

THE DISCOVERY OF DARK MATTER

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Introduction

The calculation of the galaxy rotation curve is closed to a good accuracy, the planetary orbits in the solar system and orbit of the galaxies in clusters (Zwicky first job) also is closed, there is no doubt that dark matter is the mass of the electromagnetic waves (light) emitted by the stars.

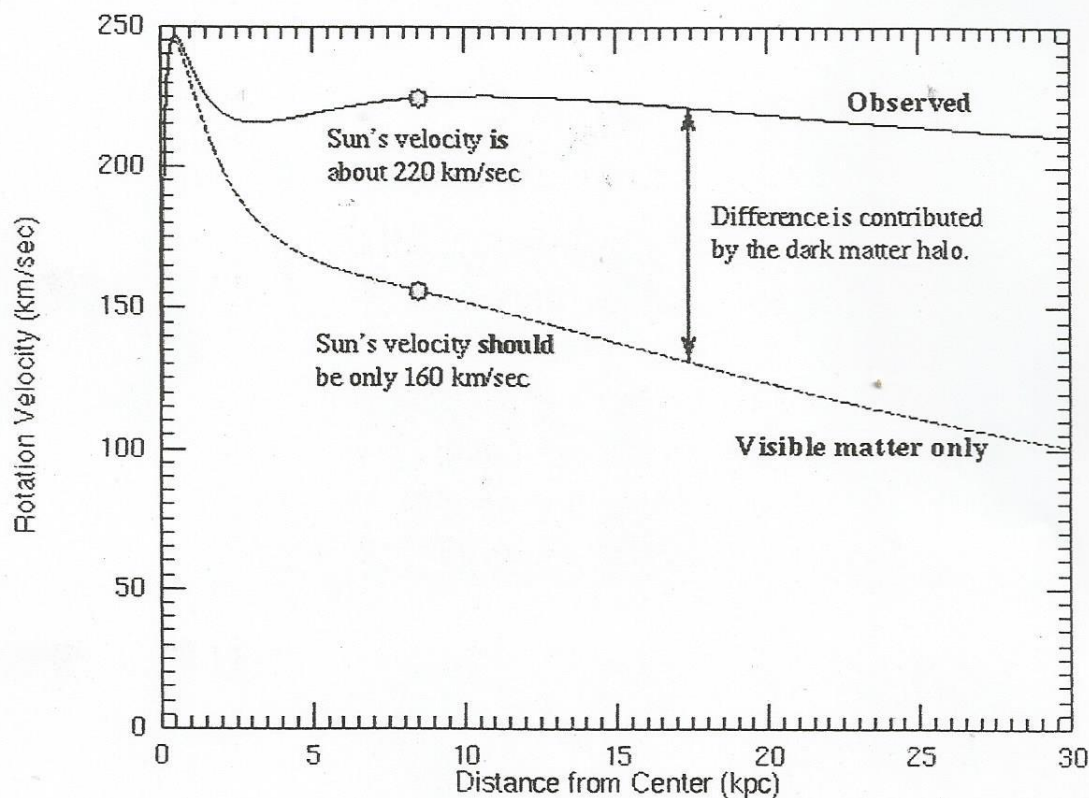
Science always say that electromagnetic waves (light) is not matter and has no mass, but this concept is superficial, not fact. For the first time in 1919 the gravitational lens were actually observed, or that the light was attracted by gravity. The Law of Universal Gravitation is a fact and indisputable concept, so the only thing left to say that the light is matter and has mass, therefore science still continued with this superficial concept that light is not matter, and agreed with the theory of relativity.

The theory of relativity has to be looked over again, and a question has to be answered: scientists know that the speed of light has no sum, but how does that happen is still not known.

So now with these calculations there is no doubt that electromagnetic waves (light) is matter and have mass, and that the reason why it is the dark matter.

The calculation of the galaxy rotation curve

Dark matter is one of the greatest mysteries of science. The first evidence of this matter is that it came from the study of galaxies orbits in the Coma Cluster from 1933 made by Zwicky, coming to the conclusion that there were 400 times more matter than he was observed. Only later on did the study of rotation of the stars in our galaxy, it was found that there was also more matter than was observed. The revolution of the planets in the solar system, has never detected dark matter, it is only noticeable in large orbits.



Above is the graph of the galaxy rotation curve (figure 1), horizontal is the distance from the galaxy center kiloparsec, one parsec equal $3,08568 \cdot 10^{16}$ m. Vertical is the speed in km / s.

The upper curve is the observed velocity of the stars, the limits of galactic disk is 15.330 Kpc, but the curve extends to 30Kpc, the speeds after 15.330Kpc is the disk gas, obtained through radio observations. The curve below is the speed that the stars and the gas would have due the quantity the matter that is observed, and the difference is due to the dark matter.

During a solar eclipse in 1919 it was seen for the first time the gravitational lens, that is, the light was attracted by the gravity, this effect was seen in Sobral in the Ceará, Brazil, and in Prince Island, on the coast of Africa.

Science always say that electromagnetic waves (light) as not being matter and having no mass, but this is a superficial concept, since the law of universal gravitation is a solid and indisputable concept, so noticing that the light is attracted by gravity, you could only say that it was matter and had mass, but science continued its superficial concept, supporting the theory of relativity.

The calculation that we present assumes that the electromagnetic waves is matter and has mass, fitting in the galaxy rotation curve. To transform energy into matter, we will use the formula $E = m.c^2$, this formula was verified experimentally by the loss of mass in radioactive substances, which is reliable.

This calculation is laborious and complex, we will have to calculate the amount of energy radiated by the stars contained in each orbit. The galactic disk has an approximate shape of a disk, the nucleus has an approximate shape of a sphere. We deduce a formula to calculate the energy contained within an orbit radiated by a homogeneous disk that emits light, its emission power will be homogeneous in watts /m² (figure 2). The galactic disk is not homogeneous, so divide it into 11 sections that are almost homogeneous, providing a good approximation. In the same way we do with the nucleus, which is almost a sphere, then we have to deduce a formula for a homogeneous sphere that emits light, its emission power will be homogeneous in watts /m³ (figure 3). The nucleus is not homogeneous, and so it will be divided into 8 sections that are almost homogeneous.

We are using the term light, which actually refers to the electromagnetic waves at all its frequencies.

Let's start from the simplest situation, the energy radiated by a point inside an orbit, this point will be positioned in the center of the orbit.

$$E = P \cdot t \quad t = \frac{D}{V_c} \quad E = \frac{P \cdot D}{V_c}$$

E – energy

P – power

t – time

D – radius of the orbit

V_c – speed of light

Let me show you an example: the energy and the light mass contained in Pluto's orbit.

Mean distance from Pluto to the sun: 5913520000 Km or 5.91352×10^{12} m

Sun power: 3.84451×10^{26} w

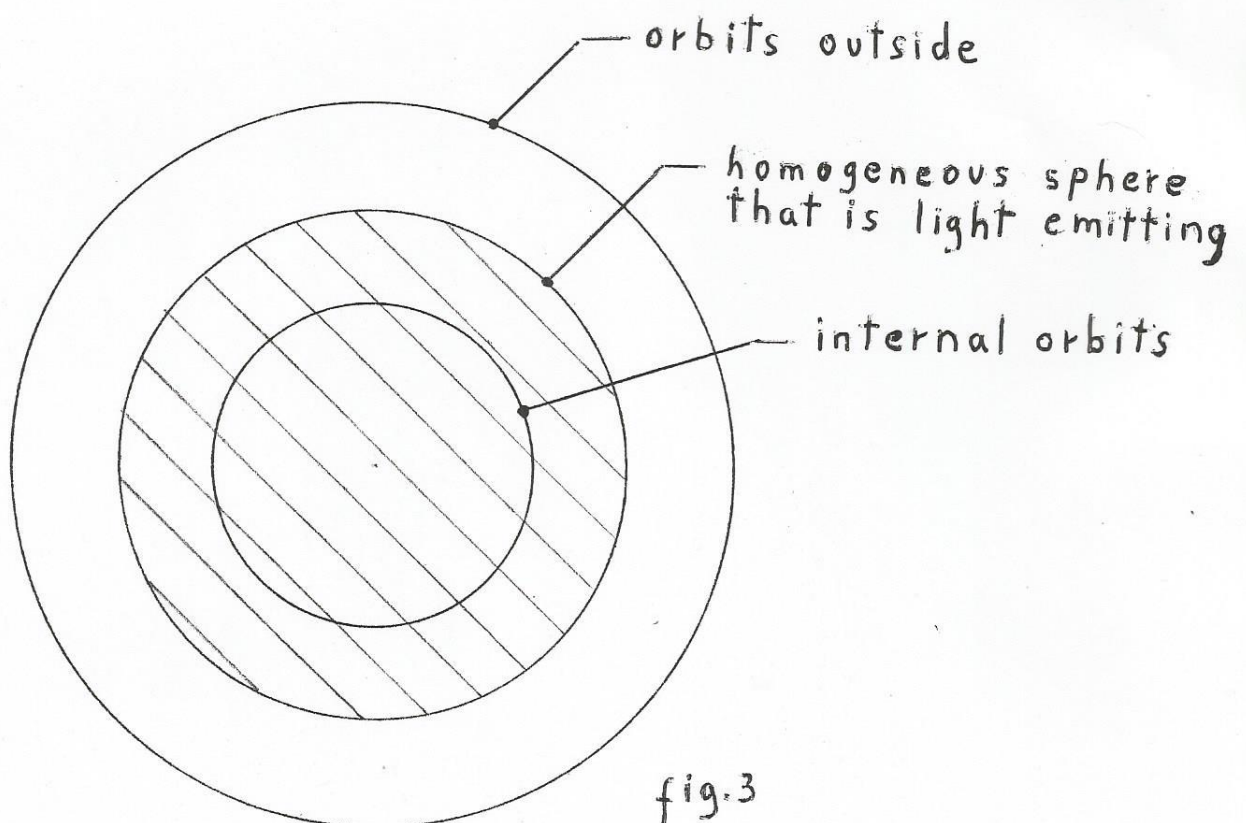
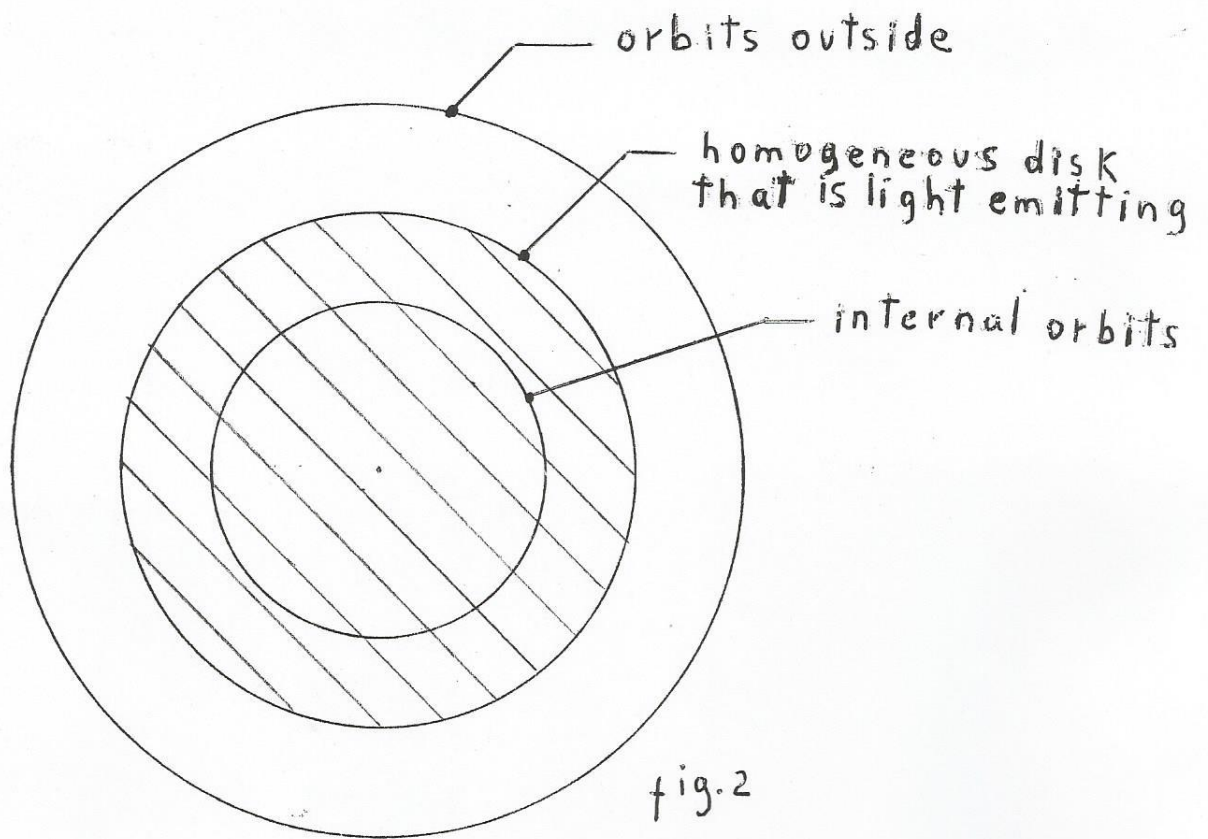
Speed of light: 299792458 m/s

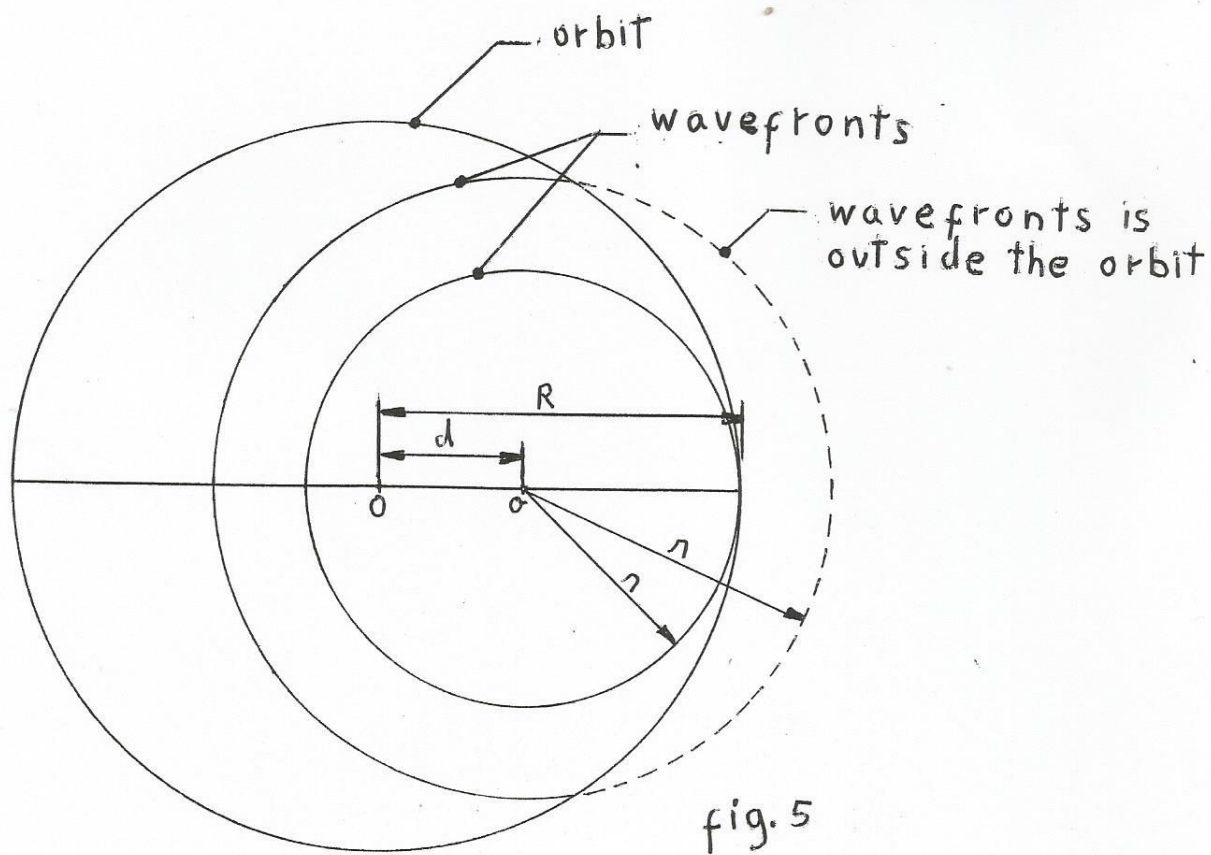
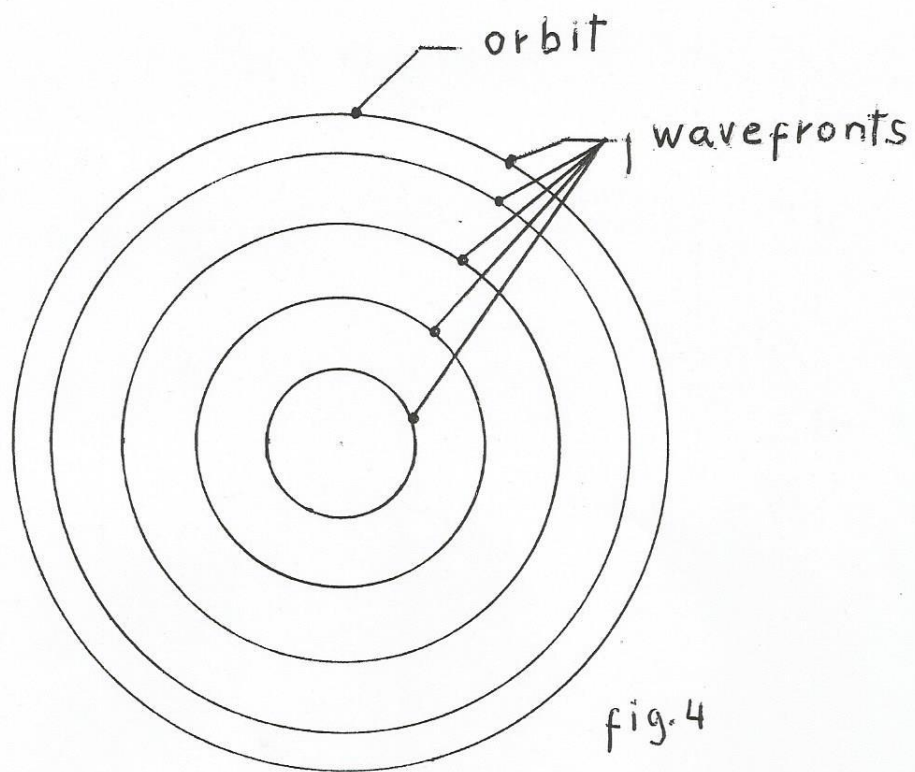
$$E = \frac{P \cdot D}{V_c} \quad E = \frac{3.84451 \times 10^{26} \times 5.91352 \times 10^{12}}{299792458} \quad E = 7.5834418 \times 10^{30} \text{ J}$$

$$E = m \cdot V_c^2 \quad m = \frac{E}{V_c^2} \quad m = \frac{7.5834418 \times 10^{30}}{299792458^2} \quad m = 8.4377169 \times 10^{13} \text{ Kg}$$

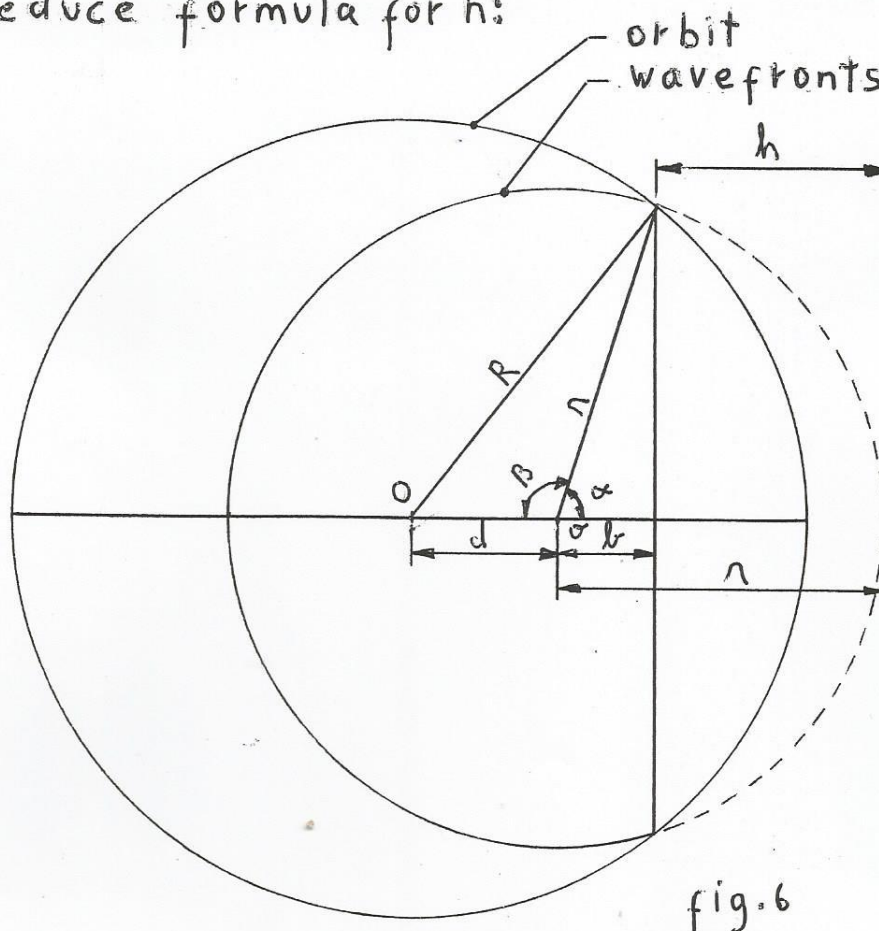
Now the first step is to deduce the formula for calculating the energy contained within the orbit (sphere) of an emitter point that is not in the center.

When the emitter point is at the center of the orbit, the wavefronts are concentric orbit (figure 4), but when the emitter point is not in the center and the radius of the wavefront is greater than R - d, part of the wavefronts is outside the orbit (Figure 5). So let's calculate the value of n (internal index), which is the division of the area of internal wavefront by total area (figure 6).





Deduce formula for n:



R – radius of the orbit

r – radius of the wavefronts

d – distance from the center orbit to the point emitter

$$n = \frac{A_{\text{sphere}} - A_{\text{external cap}}}{A_{\text{sphere}}}$$

$$\text{Area of the sphere} - 4\pi r^2$$

$$\text{Area of the cap} - 2\pi r h$$

$$R^2 = d^2 + n^2 - 2dn \cos B$$

$$\cos B = \frac{R^2 - d^2 - n^2}{-2dn}$$

$$\alpha = 180 - B$$

$$\cos B = -\cos \alpha$$

$$-\cos \alpha = \frac{R^2 - d^2 - n^2}{-2dn}$$

$$\cos \alpha = \frac{R^2 - d^2 - n^2}{2dn} \quad b = n \cos \alpha \quad b = n \cdot \frac{(R^2 - d^2 - n^2)}{2dn}$$

$$b = \frac{R^2 - d^2 - n^2}{2d} \quad h = n - b \quad h = n - \frac{(R^2 - d^2 - n^2)}{2d}$$

$$h = \frac{2dn - R^2 + d^2 + n^2}{2d}$$

$$n = \frac{A_{\text{sph}} - A_{\text{apex}}}{A_{\text{sph}}} \quad n = \frac{4\pi n^2 - 2\pi n h}{4\pi n^2}$$

$$n = \frac{2\pi A \cdot (2n - h)}{2\pi A \cdot 2n} \quad n = \frac{2n - h}{2n}$$

$$n = \frac{2n - \frac{(2dn - R^2 + d^2 + n^2)}{2d}}{2n} \quad n = \frac{4nd - 2dn + R^2 - d^2 - n^2}{4nd}$$

$$\boxed{n = \frac{2nd + R^2 - d^2 - n^2}{4nd}}$$

We will verify the formula if α is greater than 90° (figure 7).

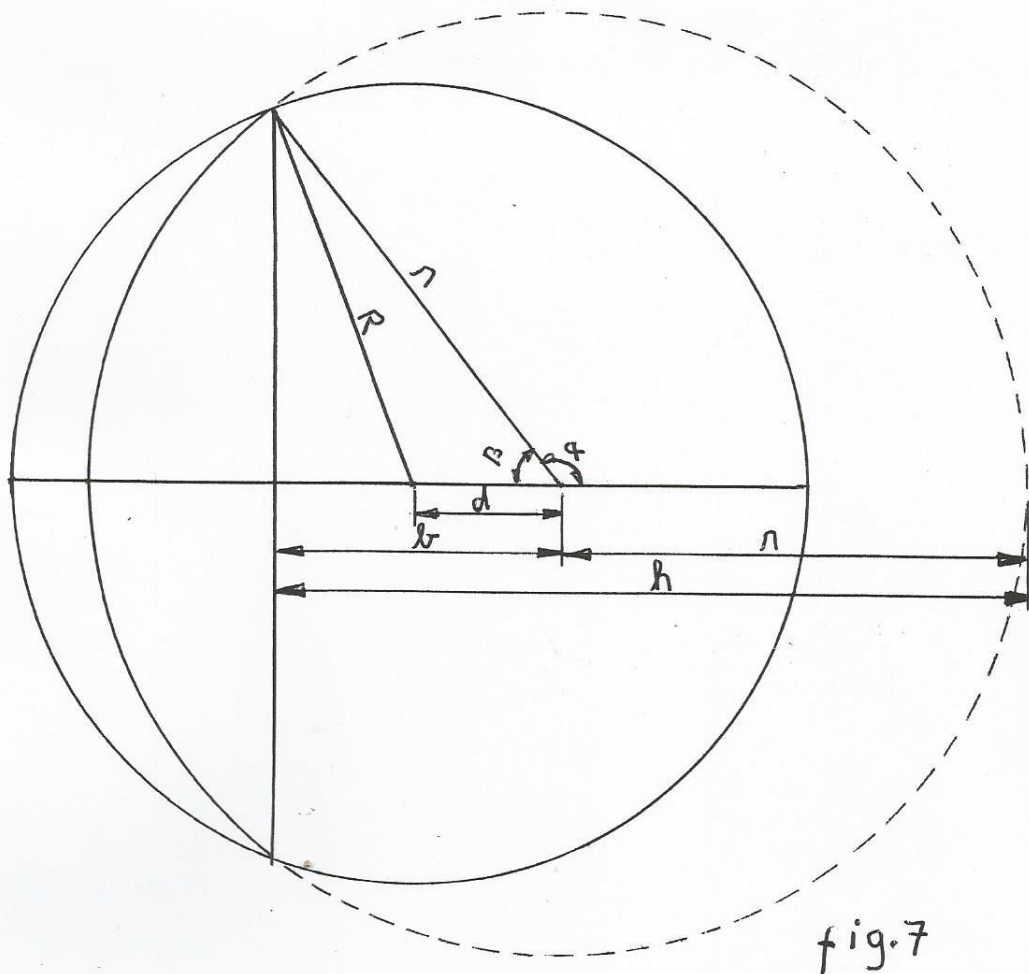


fig.7

$$R^2 = d^2 + n^2 - 2nd \cos B \quad \cos B = \frac{R^2 - d^2 - n^2}{-2nd}$$

$$\cos B = \frac{-R^2 + d^2 + n^2}{2nd} \quad b = n \cdot \cos B$$

$$b = \frac{n(-R^2 + d^2 + n^2)}{2nd} \quad b = \frac{-R^2 + d^2 + n^2}{2d}$$

$$h = n + b \quad h = n + \frac{-R^2 + d^2 + n^2}{2d}$$

$$h = \frac{2nd - R^2 + d^2 + n^2}{2d} \quad \text{The formula of } h \text{ was equal, and so is correct}$$

We now calculate the energy:

$$E = P_0 \cdot t \quad t = \frac{\eta}{V_c} \quad E = \frac{P_0 \cdot \eta}{V_c} \quad E = \frac{P_0 \cdot \eta}{V_c}$$

We call of: $K = \frac{P_0}{V_c}$, then: $E = K \cdot \eta$

P_0 = Power of the point emitter

for $0 < \eta \leq R-d$ $K = K_0 = \frac{P_0}{V_c}$

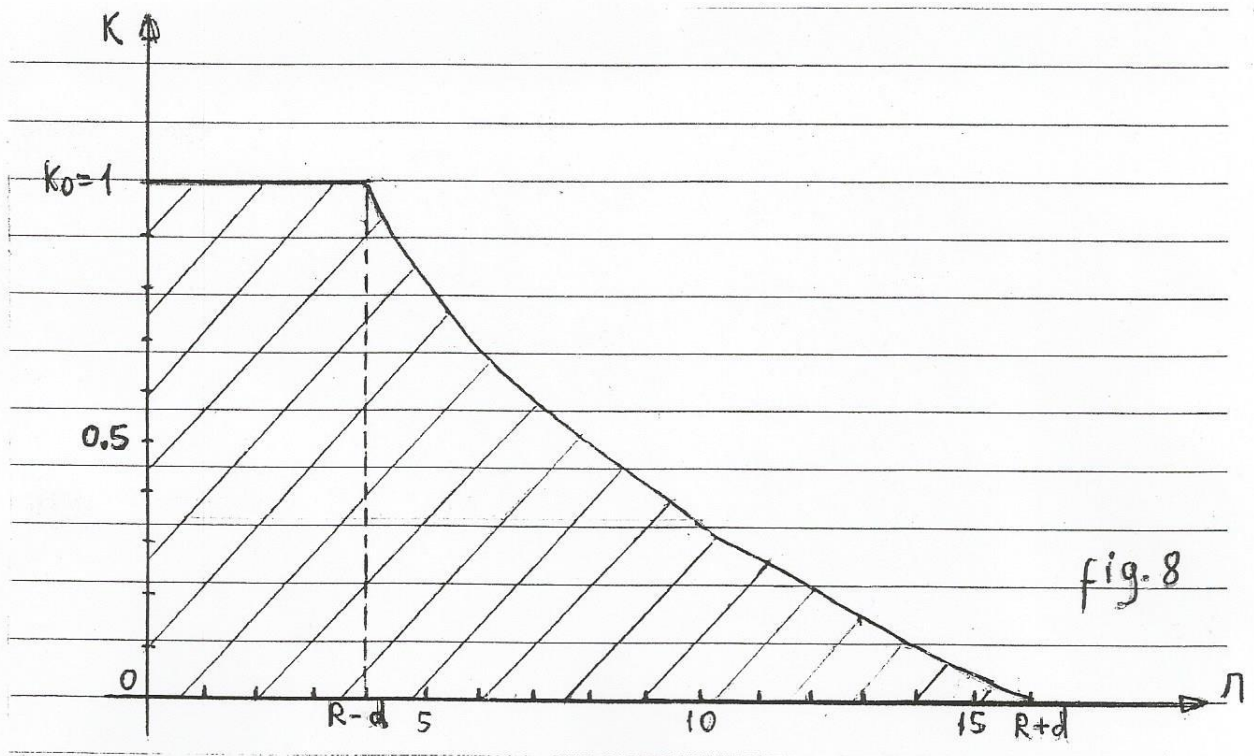
for $R-d < \eta \leq R+d$ $K = \frac{P_0 \cdot \eta}{V_c}$ $K = K_0 \cdot \eta$

$$K = K_0 \frac{(2\eta d + R^2 - d^2 - \eta^2)}{4\eta d}$$

Now let is give a numerous example:

$R=10$ $d=6$ $K_0=1$ $R-d=4$

η	K	$K = 1 \cdot \frac{(2 \cdot \eta \cdot 6 + 10^2 - 6^2 - \eta^2)}{4 \cdot \eta \cdot 6}$
4	1.00000	
5	0.82500	
6	0.69444	$K = 12\eta + 100 - 36 - \eta^2$
7	0.58929	24η
8	0.50000	
9	0.42130	$K = 12\eta + 64 - \eta^2$
10	0.35000	24η
11	0.28409	
12	0.22222	
13	0.16346	
14	0.10714	
15	0.05278	
16	0.00000	



The energy is $E = K.r$, then integrates the K in function of r between $R-d$ to $R+d$, and adding to the constant stretch of zero to $R-d$, we have the area in the graphic which is the energy (figure 8).

$$K = \frac{K_0 (2nd + R^2 - d^2 - n^2)}{4nd}$$

$$\text{onde } K_0 = \frac{P_0}{V_c}$$

$$K = K_0 \left(\frac{2nd}{4nd} + \frac{R^2}{4nd} - \frac{d^2}{4nd} - \frac{n^2}{4nd} \right)$$

$$E = K_0 \cdot \int 0,5n^0 + \left(\frac{R^2 - d^2}{4d} \right) \frac{1}{n} - \frac{n}{4d} \cdot dn$$

$$E = K_0 \cdot \left(0,5n + \left(\frac{R^2 - d^2}{4d} \right) \cdot \ln n - \frac{n^2}{8d} \right)$$

Integrating for: $r=R+d$ - $r=R-d$

$$E = K_0 \left(0.5(R+d) + \left(\frac{R^2-d^2}{4d} \right) \ln(R+d) - \frac{(R+d)^2}{8d} \right)$$

$$- K_0 \left(0.5(R-d) + \left(\frac{R^2-d^2}{4d} \right) \ln(R-d) - \frac{(R-d)^2}{8d} \right)$$

$$E = 0.5K_0R + 0.5K_0d + K_0 \ln(R+d) \left(\frac{R^2-d^2}{4d} \right) - \frac{K_0R^2}{8d} - \frac{2K_0Rd}{8d} - \frac{K_0d^2}{8d}$$

$$- 0.5K_0R + 0.5K_0d - K_0 \ln(R-d) \left(\frac{R^2-d^2}{4d} \right) + \frac{K_0R^2}{8d} - \frac{2K_0Rd}{8d} + \frac{K_0d^2}{8d}$$

$$E = K_0d + K_0 \left(\frac{R^2-d^2}{4d} \right) (\ln(R+d) - \ln(R-d)) - \frac{4K_0Rd}{8d}$$

$$E = K_0 \left(\frac{R^2-d^2}{4d} \right) (\ln(R+d) - \ln(R-d)) + K_0d - 0.5K_0R$$

to add $K_0(R-d)$ $K_0R - K_0d$

$$E = K_0 \left(\frac{R^2-d^2}{4d} \right) (\ln(R+d) - \ln(R-d)) + 0.5K_0R$$

where $K_0 = \frac{P_0}{V_c}$

formula from the point out of center

Now let's test the integration using the excel spreadsheet. Using the formula we calculate the value of E to $K_0=1$ $R=50$ $d=30$.

$$E = 1 \left(\frac{50^2 - 30^2}{4 \times 30} \right) (\ln(50-30) - \ln(50-30)) + 0.5 \times 1 \times 50$$

$$E = 43.48392$$

Now let's use the excel spreadsheet.

$$K = \frac{K_0 (2nd + R^2 - d^2 - n^2)}{4nd}$$

$$E = \frac{K_0 (2nd + R^2 - d^2 - n^2)}{4nd} \cdot dn \quad p/ \quad R-d < n < R+d$$

$$p \text{ at } a \quad 50-30 < n \leq 50+30 \quad 20 < n \leq 80 \quad dn = 0.5$$

This corresponds to 120 small pieces to be added (which is the integral concept).

$$E = 1 \times \frac{(2 \times n \times 30 + 50^2 - 30^2 - n^2)}{4 \times n \times 30} \times 0.5$$

$$E = \frac{(60n + 1600 - n^2) \times 0.5}{120n} \quad E = \frac{30n + 800 - 0.5n^2}{120n}$$

The r values is the midpoint of each segment.

$$E = n(20.25) + n(20.75) + \dots + n(79.75)$$

$$E = 0.494843107 + 0.484826807 + \dots + 0.001302900$$

$$E = 23.483599332$$

Let us now add to the constant stretch.

$$0 < \lambda \leq R-d \quad 0 < \lambda \leq 50-30 \quad 0 < \lambda \leq 20$$

$$K=K_0 \quad E=K_0 \cdot \lambda \quad E=1.20 \quad E=20$$

$$E_T = 43.48359933$$

The value is very similar to the integrated formula, so it is correct.

Below is the excel spreadsheet.

λ	E	λ	E	λ	E	λ	E
20.25	0.494843107	35.25	0.292250296	50.25	0.173294983	65.25	0.080296137
20.75	0.484826807	35.75	0.287521853	50.75	0.169904557	65.75	0.077435837
21.25	0.475183824	36.25	0.282866379	51.25	0.166539634	66.25	0.074587264
21.75	0.465888410	36.75	0.278280896	51.75	0.163199477	66.75	0.071750156
22.25	0.456917135	37.25	0.273762584	52.25	0.159883373	67.25	0.068924257
22.75	0.448248626	37.75	0.269308775	52.75	0.156590640	67.75	0.066109317
23.25	0.439863351	38.25	0.264916939	53.25	0.153320618	68.25	0.063305098
23.75	0.431743421	38.75	0.260584677	53.75	0.150072674	68.75	0.060511364
24.25	0.423872423	39.25	0.256309713	54.25	0.146846198	69.25	0.057727888
24.75	0.416235269	39.75	0.252089885	54.75	0.143640601	69.75	0.054954450
25.25	0.408818069	40.25	0.247923137	55.25	0.140455317	70.25	0.052190836
25.75	0.401608010	40.75	0.243807515	55.75	0.137289798	70.75	0.049436837
26.25	0.394593254	41.25	0.239741162	56.25	0.134143519	71.25	0.046692251
26.75	0.387762850	41.75	0.235722305	56.75	0.131015969	71.75	0.043956882
27.25	0.381106651	42.25	0.231749260	57.25	0.127906659	72.25	0.041230536
27.75	0.374615240	42.75	0.227820419	57.75	0.124815115	72.75	0.038513030
28.25	0.368279867	43.25	0.223934249	58.25	0.121740880	73.25	0.035804181
28.75	0.362092391	43.75	0.220089286	58.75	0.118683511	73.75	0.033103814
29.25	0.356045228	44.25	0.216284134	59.25	0.115642581	74.25	0.030411756
29.75	0.350131303	44.75	0.212517458	59.75	0.112617678	74.75	0.027727843
30.25	0.344344008	45.25	0.208787983	60.25	0.109608402	75.25	0.025051910
30.75	0.338677168	45.75	0.205094490	60.75	0.106614369	75.75	0.022383801
31.25	0.333125000	46.25	0.201435811	61.25	0.103635204	76.25	0.019723361
31.75	0.327682087	46.75	0.197810829	61.75	0.100670547	76.75	0.017070440
32.25	0.322343346	47.25	0.194218474	62.25	0.097720047	77.25	0.014424892
32.75	0.317104008	47.75	0.190657723	62.75	0.094783367	77.75	0.011786576
33.25	0.311959586	48.25	0.187127591	63.25	0.091860178	78.25	0.009155351
33.75	0.306905864	48.75	0.183627137	63.75	0.088950163	78.75	0.006531085
34.25	0.301938869	49.25	0.180155457	64.25	0.086053016	79.25	0.003913644
34.75	0.297054856	49.75	0.176711683	64.75	0.083168436	79.75	0.001302900
Total							23.483599330

Disk formula

To deduce the disk formula, we have to be based on the formula from the point out of center. Let's consider a very small thickness of the rim circumference (dd), all of its points will have the distance d from the center (figure 9). After integrating E_0 in function of d , we will have the sum of all the little rim, that is, the total energy of disk. This disk will have a constant power per area, then $s = P / A$ in W / m^2 .

The formula of the point out of center:

$$E = K_0 \left(\frac{R^2 - d^2}{4d} \right) (\ln(R+d) - \ln(R-d)) + 0.5 K_0 R \quad K_0 = \frac{P}{V_c}$$

$$E = \frac{P}{V_c} \left(\left(\frac{R^2 - d^2}{4d} \right) (\ln(R+d) - \ln(R-d)) + 0.5 R \right)$$

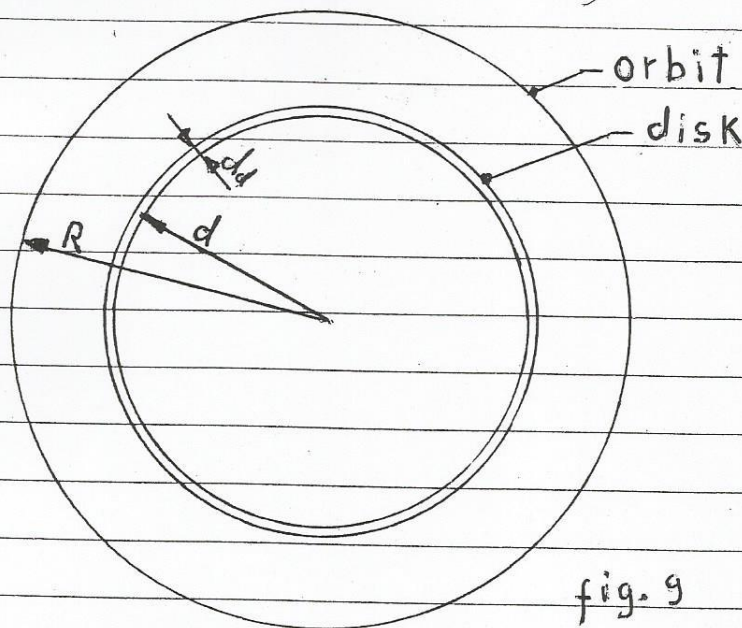


fig. 9

$$P = s \cdot A \quad P = s \cdot 2\pi d \cdot dd$$

$$E_0 = \frac{\Delta \cdot 2\pi \cdot d \cdot dd}{V_c} \left(\left(\frac{R^2 - d^2}{4d} \right) (\ln(R+d) - \ln(R-d)) + 0.5R \right)$$

$$K = \frac{\Delta \cdot 2\pi}{V_c}$$

$$E_0 = K \cdot d \cdot \left(\left(\frac{R^2 - d^2}{4d} \right) (\ln(R+d) - \ln(R-d)) + 0.5R \right) dd$$

$$E_0 = \frac{K}{4} (R^2 - d^2) ((\ln(R+d) - \ln(R-d)) + 0.5KRd) dd$$

$$E_0 = (0.25KR^2 - 0.25Kd^2) ((\ln(R+d) - \ln(R-d)) + 0.5KRd) dd$$

$$E_0 = \overset{\textcircled{1}}{0.25KR^2} \overset{\textcircled{2}}{\ln(R+d)} \overset{\textcircled{3}}{dd} - \overset{\textcircled{4}}{0.25KR^2} \overset{\textcircled{5}}{\ln(R-d)} dd - \overset{\textcircled{3}}{0.25Kd^2} \overset{\textcircled{5}}{\ln(R+d)} dd + \overset{\textcircled{4}}{0.25Kd^2} \overset{\textcircled{5}}{\ln(R-d)} dd + \overset{\textcircled{5}}{0.5KRd} dd$$

Now let's do the integration E_0 in function of d , it is a very laborious and difficult integral, so we need to do it step by step, integrating the 5 terms separately.

Term 1

$$\int_0^d 0.25KR^2 \ln(R+d) dd \quad u=R+d \quad du=dd$$

$$0.25KR^2 \int \ln u du \quad u=R+0 \quad u=R \quad u=R+d$$

$$\left. \begin{array}{l} R+d \\ | \\ 0.25KR^2(u \ln u + u) \\ | \\ R \end{array} \right\}$$

$$0.25KR^2((R+d) \ln(R+d) - (R+d)) - 0.25KR^2(R \ln R - R)$$

$$0.25KR^2(R \ln(R+d) + d \ln(R+d) - R - d) - 0.25KR^3 \ln R + 0.25KR^3$$

$$0.25KR^3 \ln(R+d) + 0.25KR^2 d \ln(R+d) - \cancel{0.25KR^3} - 0.25KR^2 d - 0.25KR^3 \ln R + \cancel{0.25KR^3}$$

$$\underline{0.25KR^3 \ln(R+d) + 0.25KR^2 d \ln(R+d) - 0.25KR^3 \ln R - 0.25KR^2 d}$$

Term 2

$$\int_0^d -0.25KR^2 \ln(R-d) dd \quad u = R-d \quad du = -dd \quad dd = -du$$

$$-0.25KR^2 \int \ln u \cdot -du \quad 0.25KR^2 \int \ln u du$$

$$u = R-d \quad u = R-0 \quad u = R \quad u = R-d$$

$$\left. \begin{array}{l} R-d \\ | \\ 0.25KR^2(u \ln u - u) \\ | \\ R \end{array} \right\}$$

$$0.25KR^2((R-d)\ln(R-d) - (R-d)) - 0.25KR^2(R\ln R - R)$$

$$0.25KR^2(R-d)\ln(R-d) - 0.25KR^2(R-d) - 0.25KR^3\ln R + 0.25KR^3$$

$$0.25KR^3\ln(R-d) - 0.25KR^2d\ln(R-d) - \cancel{0.25KR^3} + 0.25KR^2d \\ - 0.25KR^3\ln R + \cancel{0.25KR^3}$$

$$\underline{0.25KR^3\ln(R-d) - 0.25KR^2d\ln(R-d) - 0.25KR^3\ln R + 0.25KR^2d}$$

$$\int_0^d -0.25Kd^2\ln(R+d)dd \quad \begin{array}{l} \text{Term 3} \\ u=R+d \quad du=dd \\ d=u-R \end{array}$$

$$-0.25K \int_0^d (u-R)^2 \ln u du$$

$$-0.25K \int_0^d (u^2 - 2Ru + R^2) \ln u du$$

$$-0.25K \int_0^d (u^2 \ln u - 2Ru \ln u + R^2 \ln u) du$$

Formula of the integral: $\int x^m \ln x dx = x^{m+1} \left(\frac{\ln x}{m+1} - \frac{1}{(m+1)^2} \right)$

$$-0.25K \left(u^3 \left(\frac{\ln u}{3} - \frac{1}{9} \right) - 2Ru^2 \left(\frac{\ln u}{2} - \frac{1}{4} \right) + \right.$$

$$R^2 (u \ln u - u)$$

$$- 0.25K \left(\frac{u^3 \ln u}{3} - \frac{u^3}{9} - Ru^2 \ln u + 0.5Ru^2 + R^2u \ln u - R^2u \right)$$

R+d

$$\left| \begin{array}{l} -0.25K \frac{u^3 \ln u}{3} + \frac{0.25K u^3}{9} + 0.25KRu^2 \ln u - 0.125KRu^2 \\ R \end{array} \right.$$

$$- 0.25KR^2u \ln u + 0.25KR^2u$$

$$u=R+d \quad u=R+0 \quad u=R \quad u=R+d$$

$$- \frac{0.25K(R+d)^3 \ln(R+d)}{3} + \frac{0.25K(R+d)^3}{9}$$

$$+ 0.25KR(R+d)^2 \ln(R+d) - 0.125KR(R+d)^2$$

$$- 0.25KR^2(R+d) \ln(R+d) + 0.25KR^2(R+d)$$

$$- \left(- \frac{0.25KR^3 \ln R}{3} + \frac{0.25KR^3}{9} + 0.25KR^2 \ln R \right.$$

$$\left. - 0.125KR^2 - 0.25KR^2R \ln R + 0.25KR^2R \right) \therefore$$

$$- \frac{0.25K(R+d)^3 \ln(R+d)}{3} + \frac{0.25K(R+d)^3}{9}$$

$$+ 0.25KR(R+d)^2 \ln(R+d) - 0.125KR(R+d)^2$$

$$-0.25KR^2(R+d)\ln(R+d) + 0.25KR^2(R+d)$$

$$+ \frac{0.25KR^3\ln R}{3} - \frac{0.25KR^3}{9} - \cancel{0.25KR^3\ln R} + 0.125KR^3$$

$$+ \cancel{0.25KR^3\ln R} - 0.25KR^3 \therefore$$

$$\frac{-0.25K(R+d)^3\ln(R+d)}{3} + \frac{0.25K(R+d)^3}{9}$$

$$+ 0.25KR(R+d)^2\ln(R+d) - 0.125KR(R+d)^2 \\ - 0.25KR^2(R+d)\ln(R+d) + 0.25KR^2(R+d) + \frac{0.25KR^3\ln R}{3}$$

$$\frac{-2.75KR^3}{18} \therefore$$

Resolving the polynomials $(R+d)^3$, $(R+d)^2$ and resolving this long algebraic expression results in:

$$0.25K\ln(R+d) \left(\frac{-R^3}{3} - \frac{d^3}{3} \right) + 0.25K \left(\frac{R^2d}{3} - \frac{Rd^2}{6} + \frac{d^3}{9} \right) \\ + \frac{0.25KR^3\ln R}{3}$$

Term 4

$$\int_0^d 0.25Kd^2\ln(R-d)dd \quad \begin{array}{l} u=R-d \quad du=-dd \quad dd=-du \\ -d=u-R \quad d=R-u \end{array}$$

$$\begin{aligned}
 & 0.25K \int_0^d (R-u)^2 \ln u \cdot (-du) - 0.25K \int_0^d (R^2 - 2Ru + u^2) \ln u \, du \\
 & - 0.25K \int_0^d (u^2 \ln u - 2Ru \ln u + R^2 \ln u) \, du \\
 & \text{Equal to the term 3}
 \end{aligned}$$

$$\begin{aligned}
 & \left| \begin{array}{l} R-d \\ R \end{array} \right| - \frac{0.25K u^3 \ln u}{3} + \frac{0.25K u^3}{9} + 0.25KR u^2 \ln u \\
 & - 0.125KR u^2 - 0.25KR^2 u \ln u + 0.25KR^2 u
 \end{aligned}$$

$$u = R-d \quad u = R-0 \quad u = R \quad u = R-d$$

$$- \frac{0.25K (R-d)^3 \ln(R-d)}{3} + \frac{0.25K (R-d)^3}{9} + 0.25KR (R-d)^2 \ln(R-d)$$

$$- 0.125KR (R-d)^2 - 0.25KR^2 (R-d) \ln(R-d) + 0.25KR^2 (R-d)$$

$$- \left(- \frac{0.25KR^3 \ln R}{3} + \frac{0.25KR^3}{9} + 0.25KR R^2 \ln R \right.$$

$$\left. - 0.125KR R^2 - 0.25KR^2 R \ln R + 0.25KR^2 R \right) \therefore$$

$$- \frac{0.25K (R-d)^3 \ln(R-d)}{3} + \frac{0.25K (R-d)^3}{9} + 0.25KR (R-d)^2 \ln(R-d)$$

$$-0.125KR(R-d)^2 - 0.25KR^2(R-d)\ln(R-d) + 0.25KR^2(R-d)$$

$$+ \frac{0.25KR^3\ln R}{3} - \frac{0.25KR^3}{9} - \cancel{0.25KR^3\ln R} + 0.125KR^3$$

$$+ \cancel{0.25KR^3\ln R} - 0.25KR^3 \therefore$$

$$- \frac{0.25K(R-d)^3\ln(R-d)}{3} + \frac{0.25K(R-d)^3}{9} + 0.25KR(R-d)^2\ln(R-d)$$

$$-0.125KR(R-d)^2 - 0.25KR^2(R-d)\ln(R-d) + 0.25KR^2(R-d)$$

$$+ \frac{0.25KR^3\ln R}{3} - \frac{2.75KR^3}{18} \therefore$$

Resolving the polynomials $(R-d)^3$, $(R-d)^2$ and resolving this long algebraic expression results in:

$$0.25K\ln(R-d) \left(-\frac{R^3}{3} + \frac{d^3}{3} \right) + 0.25K \left(-\frac{R^2d}{3} - \frac{Rd^2}{6} - \frac{d^3}{9} \right)$$

$$+ \frac{0.25KR^3\ln R}{3}$$

$$\int_0^d 0.5KRd \, dd \quad \text{Term 5} \quad \frac{0.5KRd^2}{2} \quad \underline{0.25KRd^2}$$

Adding the 5 terms

$$E = 0.25K \ln(R+d) \left(R^3 + R^2d - \frac{R^3}{3} - \frac{d^3}{3} \right)$$

$$+ 0.25K \ln(R-d) \left(R^3 - R^2d - \frac{R^3}{3} + \frac{d^3}{3} \right)$$

$$+ 0.25K \ln R \left(-R^3 - R^3 + \frac{R^3}{3} + \frac{R^3}{3} \right)$$

$$+ 0.25K \left(-R^2d + R^2d + \frac{R^2d}{3} - \frac{Rd^2}{6} + \frac{d^3}{9} - \frac{R^2d}{3} - \frac{Rd^2}{6} - \frac{d^3}{9} + Rd^2 \right)$$

$$E = 0.25K \ln(R+d) \left(\frac{2}{3} R^3 + R^2d - \frac{d^3}{3} \right)$$

$$+ 0.25K \ln(R-d) \left(\frac{2}{3} R^3 - R^2d + \frac{d^3}{3} \right) - \frac{KR^3 \ln R}{3} + \frac{KRd^2}{6}$$

$$\text{para } K = \frac{2\pi\epsilon}{V_c}$$

$$E = \frac{\pi\epsilon}{6V_c} \ln(R+d) \left(2R^3 + 3R^2d - d^3 \right)$$

$$+ \frac{\pi \Delta \ln(R-d)}{6V_c} (2R^3 + 3R^2d + d^3) - \frac{2\pi \Delta R^3 \ln R}{3V_c} + \frac{\pi \Delta R d^2}{3V_c}$$

$$E = \frac{\pi \Delta}{3V_c} \left[0.5 \ln(R+d) (2R^3 + 3R^2d - d^3) + 0.5 \ln(R-d) (2R^3 - 3R^2d + d^3) - 2R^3 \ln R + R d^2 \right] \text{ Formula of the internal disk } d \leq R$$

Now let's test the integration using the excel spreadsheet. Using the formula we calculate the value of E to $\frac{\pi \Delta}{3V_c} = 1$ $R=50$ $d=30$.

$$E = 1 \left[0.5 \times \ln(50+30) (2 \times 50^3 + 3 \times 50^2 \times 30 - 30^3) + 0.5 \times \ln(50-30) (2 \times 50^3 - 3 \times 50^2 \times 30 + 30^3) - 2 \times 50^3 \times \ln 50 + 50 \times 30^2 \right]$$

$$E = 126457.2539$$

Now let's use the excel spreadsheet.

$$E_0 = \frac{K(R^2 - d^2)}{4} \left(\ln(R+d) - \ln(R-d) \right) + 0.5 K R d$$

$$\text{para } \frac{\pi \Delta}{3V_c} = 1 \quad \frac{\pi \Delta}{V_c} = 3 \quad \frac{\Delta 2\pi}{V_c} = 6 \quad K=6$$

$$R=50 \quad d=30 \quad dd=0,2$$

This corresponds to 150 small pieces to be added (which is the integral concept).

$$E_0 = \frac{6(50^2 - d^2)}{4} (\ln(50+d) - \ln(50-d) + 0.5 \times 6 \times 50 \times d) \times 0.2$$

$$E_0 = 0.3(2500 - d^2)(\ln(50+d) - \ln(50-d)) + 30d$$

The d values is the midpoint of each segment.

$$E = d(29.9) + d(29.7) + \dots + d(0.1)$$

$$E = 1561.906859 + 1554.819752 + \dots + 5.999992$$

$$E = 126457.4619$$

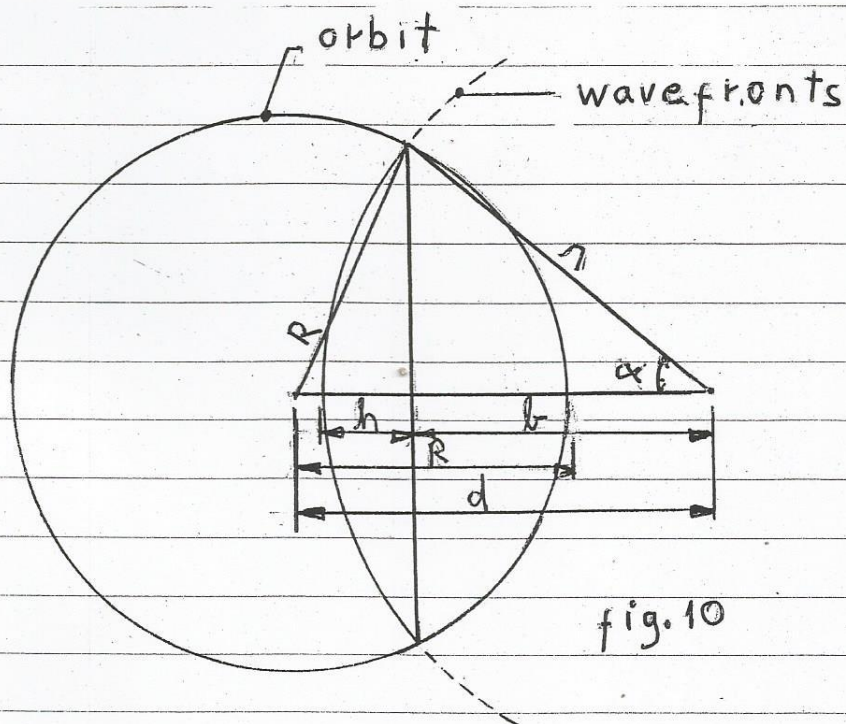
The value is very similar to the integrated formula, so it is correct.

Below is the excel spreadsheet.

d	E ₀	d	E ₀	d	E ₀	d	E ₀
29.9	1561.906859	20.7	1168.411882	11.5	677.701266	2.3	137.902623
29.7	1554.819752	20.5	1158.577465	11.3	666.336251	2.1	125.925886
29.5	1547.655764	20.3	1148.698481	11.1	654.948766	1.9	113.945112
29.3	1540.415805	20.1	1138.775484	10.9	643.539236	1.7	101.960687
29.1	1533.100772	19.9	1128.809022	10.7	632.108085	1.5	89.972995
28.9	1525.711548	19.7	1118.799638	10.5	620.655735	1.3	77.982422
28.7	1518.249007	19.5	1108.747874	10.3	609.182608	1.1	65.989351
28.5	1510.714009	19.3	1098.654265	10.1	597.689124	0.9	53.994168
28.3	1503.107401	19.1	1088.519343	9.9	586.175699	0.7	41.997256
28.1	1495.430020	18.9	1078.343637	9.7	574.642752	0.5	29.999000
27.9	1487.682693	18.7	1068.127671	9.5	563.090696	0.3	17.999784
27.7	1479.866235	18.5	1057.871966	9.3	551.519947	0.1	5.999992
27.5	1471.981449	18.3	1047.577040	9.1	539.930916	Total E=126457.461900	

27.3	1464.029131	18.1	1037.243406	8.9	528.324016
27.1	1456.010065	17.9	1026.871574	8.7	516.699656
26.9	1447.925025	17.7	1016.462051	8.5	505.058245
26.7	1439.774777	17.5	1006.015342	8.3	493.400192
26.5	1431.560077	17.3	995.531945	8.1	481.725902
26.3	1423.281671	17.1	985.012359	7.9	470.035781
26.1	1414.940298	16.9	974.457077	7.7	458.330234
25.9	1406.536689	16.7	963.866590	7.5	446.609664
25.7	1398.071563	16.5	953.241386	7.3	434.874474
25.5	1389.545635	16.3	942.581951	7.1	423.125064
25.3	1380.959609	16.1	931.888766	6.9	411.361836
25.1	1372.314184	15.9	921.162311	6.7	399.585188
24.9	1363.610049	15.7	910.403062	6.5	387.795520
24.7	1354.847888	15.5	899.611493	6.3	375.993229
24.5	1346.028375	15.3	888.788076	6.1	364.178712
24.3	1337.152179	15.1	877.933280	5.9	352.352365
24.1	1328.219962	14.9	867.047570	5.7	340.514584
23.9	1319.232378	14.7	856.131410	5.5	328.665762
23.7	1310.190077	14.5	845.185261	5.3	316.806295
23.5	1301.093699	14.3	834.209583	5.1	304.936574
23.3	1291.943881	14.1	823.204833	4.9	293.056993
23.1	1282.741252	13.9	812.171463	4.7	281.167943
22.9	1273.486436	13.7	801.109927	4.5	269.269815
22.7	1264.180051	13.5	790.020675	4.3	257.363000
22.5	1254.822708	13.3	778.904154	4.1	245.447888
22.3	1245.415015	13.1	767.760810	3.9	233.524869
22.1	1235.957572	12.9	756.591086	3.7	221.594331
21.9	1226.450975	12.7	745.395425	3.5	209.656663
21.7	1216.895815	12.5	734.174267	3.3	197.712253
21.5	1207.292677	12.3	722.928048	3.1	185.761488
21.3	1197.642142	12.1	711.657205	2.9	173.804757
21.1	1187.944784	11.9	700.362173	2.7	161.842444
20.9	1178.201176	11.7	689.043383	2.5	149.874937

The disk formula was developed for when the orbit is outside the disk. Now we must check the situation in which the orbit is inside the disk. We deduce the formula for the part of the disk that is outside the orbit and then will add to part of the disk that is equal to orbit. The steps are similar to the internal disk, we start by calculating the energy within the orbit due to an external emitter point (figure 10).



$$R^2 = d^2 + n^2 - 2dn \cos \alpha \quad \cos \alpha = \frac{R^2 - d^2 - n^2}{-2dn} \quad \cos \alpha = \frac{d^2 + n^2 - R^2}{2dn}$$

$$h = n \cdot \cos \alpha \quad h = n - b \quad h = n - n \cdot \cos \alpha$$

$$h = n - \frac{d^2 + n^2 - R^2}{2d} \quad h = n - \frac{(d^2 + n^2 - R^2)}{2d}$$

$$h = \frac{2dn - d^2 - n^2 + R^2}{2d} \quad n = \frac{\text{Area cap}}{\text{Area sphere}} \quad n = \frac{2\pi n \cdot h}{4\pi n^2}$$

$$n = \frac{h}{2n} \quad n = \frac{2dn - d^2 - n^2 + R^2}{2d \cdot 2n} \quad \boxed{n = \frac{2nd + R^2 - d^2 - n^2}{4nd}}$$

We now calculate the energy

$$E = P \cdot t \quad t = \frac{n}{V_c} \quad E = P \cdot \frac{n}{V_c} \quad E = \frac{P \cdot n}{V_c}$$

We call of: $K = \frac{P}{V_c}$, Then: $E = K \cdot r$

P_0 = Power of the point emitter

for $0 < r < d - R$ K will be negative (cancel)

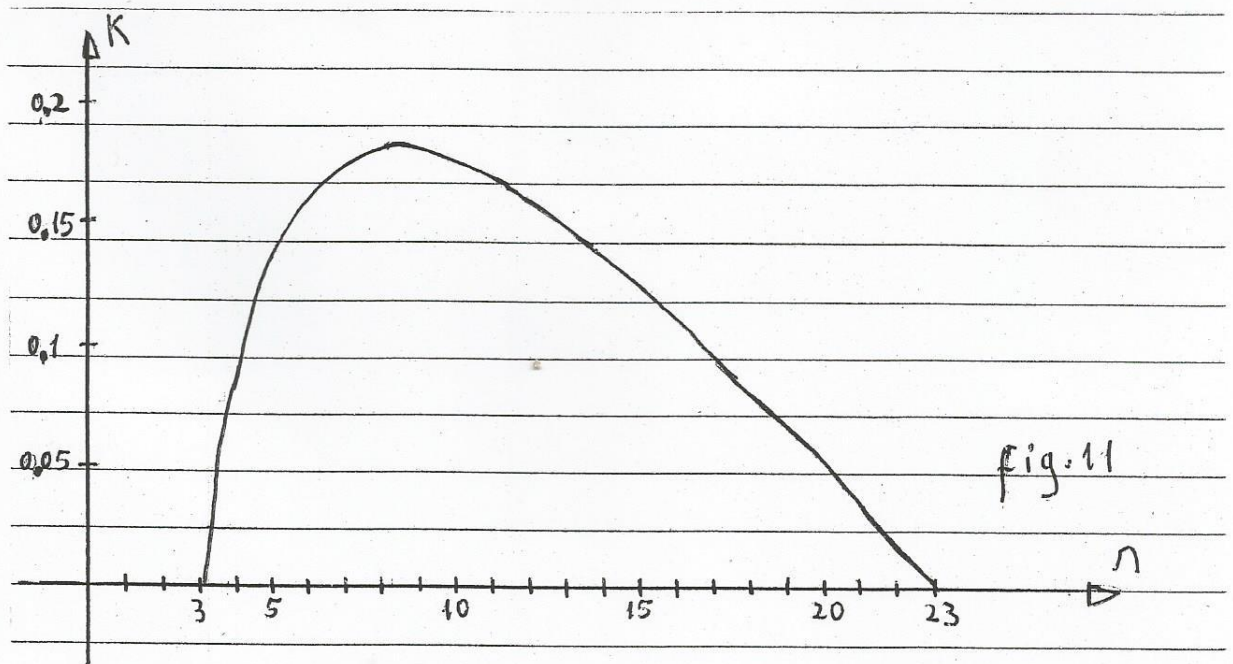
para $d - R \leq r \leq d + R$ K will be positive (consider)

$$K = \frac{P_0 \times r}{V_c} \quad K = K_0 \times r \quad K = K_0 \frac{(2rd + R^2 - d^2 - r^2)}{4rd}$$

Now let is give a numerous example $R=10$ $d=13$ $K_0=1$ and a graph (figure 11).

$$K = 1 \frac{(2 \times r \times 13 + 10^2 - 13^2 - r^2)}{4 \times r \times 13} \quad K = \frac{26r - r^2 - 69}{52r}$$

r	K	r	K
0.5	-2.163462	11	0.167832
1	-0.846154	12	0.158654
1.5	-0.413462	13	0.147929
2	-0.201923	14	0.135989
2.5	-0.078846	15	0.123077
3	0	16	0.109375
4	0.091346	17	0.095023
5	0.138462	18	0.080128
6	0.163462	19	0.064777
7	0.175824	20	0.049038
8	0.180288	21	0.032967
9	0.179487	22	0.016608
10	0.175000	23	0



The formula for K in function of r was identical to the previous one (the point out of center), then the integration will be the same, the interval will be different, it will be from $d - R$ to $d + R$.

$$E = K_0 \left(0.5n + \left(\frac{R^2 - d^2}{4d} \right) \cdot \ln n - \frac{n^2}{8d} \right)$$

integrando de $n = d + R$ - $n = d - R$

$$E = K_0 \left(0.5(d+R) + \left(\frac{R^2 - d^2}{4d} \right) \cdot \ln(d+R) - \frac{(d+R)^2}{8d} \right)$$

$$- K_0 \left(0.5(d-R) + \left(\frac{R^2 - d^2}{4d} \right) \cdot \ln(d-R) - \frac{(d-R)^2}{8d} \right) \therefore$$

$$E = \cancel{0.5K_0d} + 0.5K_0R + K_0 \ln(d+R) \frac{(R^2 - d^2)}{4d} - \cancel{\frac{K_0d^2}{8d}} - \frac{2K_0dR}{8d}$$

$$-\frac{K_0 R^2}{8d} - \cancel{0.5 K_0 d} + 0.5 K_0 R - K_0 \ln(d-R) \frac{(R^2-d^2)}{4d} + \cancel{\frac{K_0 d^2}{8d}}$$

$$-\frac{2 K_0 d R}{8d} + \cancel{\frac{K_0 R^2}{8d}}$$

$$E = K_0 R + K_0 \frac{(R^2-d^2)}{4d} (\ln(d+R) - \ln(d-R)) - \frac{4 K_0 d R}{8d}$$

$$E = K_0 \left(\frac{R^2-d^2}{4d} \right) (\ln(d+R) - \ln(d-R)) + K_0 R - 0.5 K_0 R$$

$$E = K_0 \left(\frac{R^2-d^2}{4d} \right) (\ln(d+R) - \ln(d-R)) + 0.5 K_0 R \quad \text{onde } K_0 = \frac{P}{V_c}$$

fórmula do ponto externo

Now let's test the integration using the excel spreadsheet. Using the formula we calculate the value of E to $K_0=1$ $R=50$ $d=65$.

$$E = 1 \left(\frac{50^2 - 65^2}{4 \times 65} \right) (\ln(65+50) - \ln(65-50)) + 0.5 \times 1 \times 50$$

$$E = 11.48607183$$

Now let's use the excel spreadsheet.

$$E = K_0 \frac{(2\pi d + R^2 - d^2 - r^2)}{4\pi d} \times dr$$

$$p/ \quad d-R \leq r \leq d+R \quad 15 \leq r \leq 115 \quad dr=1$$

This corresponds to 100 small pieces to be added (which is the integral concept).

$$E = \frac{1}{4 \times 1 \times 65} (2 \times 1 \times 65 + 50^2 - 65^2 - 1^2) \times 1 \quad E = \frac{1301 - 1^2 - 1725}{2601}$$

The r values is the midpoint of each segment.

$$E = 1(15.5) + 1(16.5) + \dots + 1(114.5)$$

$$E = 0.012344913 + 0.034440559 + \dots + 0.001671145$$

$$E = 11.48727861$$

The value is very similar to the integrated formula, so it is correct.

Below is the excel spreadsheet.

Δ	E	Δ	E
15.5	0.012344913	65.5	0.146785085
16.5	0.034440559	66.5	0.144462117
17.5	0.053571429	67.5	0.142094017
18.5	0.070218295	68.5	0.139682762
19.5	0.084763314	69.5	0.137230216
20.5	0.097514071	70.5	0.134738134
21.5	0.108720930	71.5	0.132208176
22.5	0.118589744	72.5	0.129641910
23.5	0.127291326	73.5	0.127040816
24.5	0.134968603	74.5	0.124406298
25.5	0.141742081	75.5	0.121739684
26.5	0.147714078	76.5	0.119042232
27.5	0.152972028	77.5	0.116315136
28.5	0.157591093	78.5	0.113559530
29.5	0.161636245	79.5	0.110776488
30.5	0.165163934	80.5	0.107967033
31.5	0.168223443	81.5	0.105132138
32.5	0.170857988	82.5	0.102272727
33.5	0.173105626	83.5	0.099389682
34.5	0.175000000	84.5	0.096483842
35.5	0.176570964	85.5	0.093556005
36.5	0.177845100	86.5	0.090606936
37.5	0.178846154	87.5	0.087637363

38.5	0.179595405	88.5	0.084647979
39.5	0.180111977	89.5	0.081639450
40.5	0.180413105	90.5	0.078612410
41.5	0.180514365	91.5	0.075567465
42.5	0.180429864	92.5	0.072505198
43.5	0.180172414	93.5	0.069426162
44.5	0.179753673	94.5	0.066330891
45.5	0.179184277	95.5	0.063219895
46.5	0.178473945	96.5	0.060093663
47.5	0.177631579	97.5	0.056952663
48.5	0.176665345	98.5	0.053797345
49.5	0.175582751	99.5	0.050628141
50.5	0.174390708	100.5	0.047445465
51.5	0.173095594	101.5	0.044249716
52.5	0.171703297	102.5	0.041041276
53.5	0.170219267	103.5	0.037820513
54.5	0.168648553	104.5	0.034587781
55.5	0.166995842	105.5	0.031343420
56.5	0.165265487	106.5	0.028087757
57.5	0.163461538	107.5	0.024821109
58.5	0.161587771	108.5	0.021543779
59.5	0.159647705	109.5	0.018256059
60.5	0.157644628	110.5	0.014958232
61.5	0.155581614	111.5	0.011650569
62.5	0.153461538	112.5	0.008333333
63.5	0.151287099	113.5	0.005006777
64.5	0.149060823	114.5	0.001671145

Total: 11.487278610

We now deduce the energy that the outside disk to orbit radiates inside of the orbit, we will use the same previous procedures (figure 12).

$$E = k_0 \left(\frac{R^2 - d^2}{4d} \right) (\ln(d+R) - \ln(d-R)) + 0.5 k_0 R$$

onde $k_0 = \frac{P}{V_c}$

$$E = \frac{P}{V_c} \left(\left(\frac{R^2 - d^2}{4d} \right) (\ln(d+R) - \ln(d-R)) + 0.5 R \right)$$

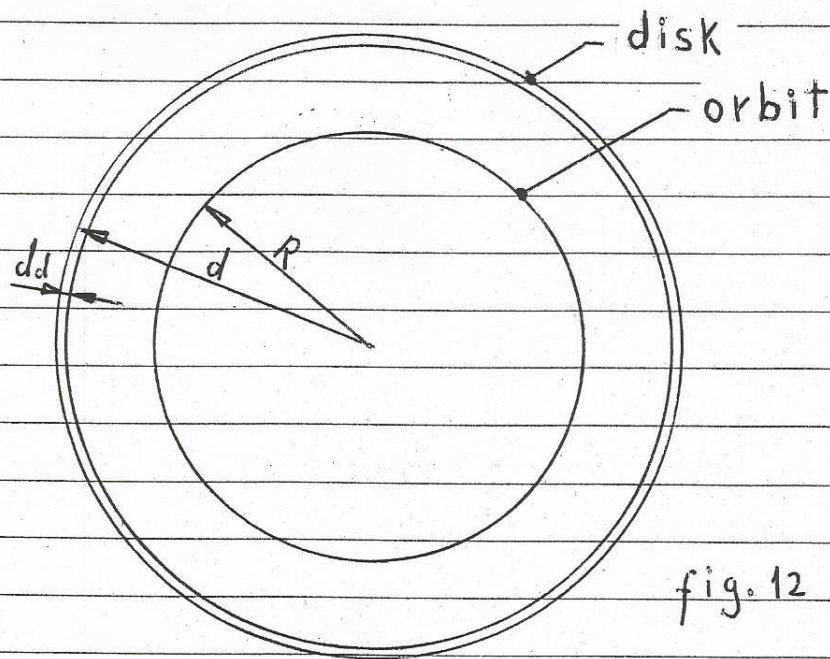


fig. 12

$$P = \lambda \cdot A \quad P = \lambda \cdot 2\pi d \cdot dd$$

$$E_0 = \frac{\lambda 2\pi d \cdot dd}{V_c} \left(\left(\frac{R^2 - d^2}{4d} \right) (\ln(R+d) - \ln(d-R)) + 0.5R \right)$$

$$K = \frac{\lambda 2\pi}{V_c}$$

$$E_0 = K d \left(\left(\frac{R^2 - d^2}{4d} \right) (\ln(R+d) - \ln(d-R)) + 0.5K \right) dd$$

$$E_0 = K \frac{(R^2 - d^2)}{4} ((\ln(R+d) - \ln(d-R)) + 0.5KRd) dd$$

$$E_0 = (0.25KR^2 - 0.25Kd^2)((\ln(R+d) - \ln(d-R)) + 0.5KRd) dd$$

$$E_0 = 0.25KR^2 \ln(R+d) dd - 0.25KR^2 \ln(d-R) dd$$

$$- 0.25Kd^2 \ln(R+d) dd + 0.25Kd^2 \ln(d-R) dd + 0.5KRd dd$$

The terms 1,3 and 5 are identical, the integral will be the same, the integration interval is different from R to d.

Term 1

intervalo R até d integração já feita

$$u = R+d \quad p/d = R \quad u = R+R \quad u = 2R$$

$$u = R+d \quad p/d = d \quad u = R+d$$

$$\int_{2R}^{R+d} 0.25 \cdot K \cdot R^2 (u \ln u - u)$$

$$0.25 K R^2 ((R+d) \ln(R+d) - (R+d)) - 0.25 K R^2 (2R \ln 2R - 2R) \therefore$$

$$0.25 K R^2 (R \ln(R+d) + d \ln(R+d) - R - d) - 0.5 K R^3 \ln 2R + 0.5 K R^3 \therefore$$

$$0.25 K R^3 \ln(R+d) + 0.25 K R^2 d \ln(R+d) - 0.25 K R^3 - 0.25 K R^2 d - 0.5 K R^3 \ln 2R + 0.5 K R^3 \therefore$$

$$0.25 K R^3 \ln(R+d) + 0.25 K R^2 d \ln(R+d) - 0.25 K R^2 d - 0.5 K R^3 \ln 2R + 0.25 K R^3$$

Term 2

$$\int_R^d -0.25 \times K \times R^2 \ln(d-R) dd \quad u = d-R \quad du = dd \quad dd = du$$

$$-0.25 \times K \times R^2 \int_R^d \ln u \cdot du \quad u = d-R \quad p/d = R \quad u = R-R \quad u = 0$$

$$u = d-R \quad p/d = d \quad u = d-R$$

$$\int_0^{d-R} -0.25 \times K \times R^2 (u \ln u - u)$$

$$-0.25 K R^2 ((d-R) \ln(d-R) - (d-R)) - (-0.25 K R^2 (0 \ln 0 - 0)) \therefore$$

$$-0.25 K R^2 (d \ln(d-R) - R \ln(d-R) - d + R) \therefore$$

$$-0.25 K R^2 d \ln(d-R) + 0.25 K R^3 \ln(d-R) + 0.25 K R^2 d - 0.25 K R^3$$

Term 3

Interval from R to d, integration already done

$$\begin{array}{l} u=R+d \quad p/ \quad d=R \quad u=R+R \quad u=2R \\ u=R+d \quad p/ \quad d=d \quad u=R+d \end{array}$$

$$\int_{2R}^{R+d} \left(-\frac{0.25 K u^3 \ln u}{3} + \frac{0.25 K u^3}{9} + 0.25 K R u^2 \ln u - 0.125 K R u^2 \right)$$

$$-0.25 K R^2 u \ln u + 0.25 K R^2 u \therefore$$

(R + d) part has been made, making only the part of 2R

$$\frac{0.25 K 8 R^3 \ln 2R}{3} - \frac{0.25 K 8 R^3}{9} - 0.25 K R 4 R^2 \ln 2R + 0.125 K R 4 R^2$$

$$+ 0.25 K R^2 2R \ln 2R - 0.25 K R^2 2R \therefore$$

$$\frac{2KR^3 \ln 2R}{3} - \frac{2KR^3}{9} - KR^3 \ln 2R + 0.5KR^3 + 0.5KR^3 \ln 2R - 0.5KR^3 \therefore$$

$$\frac{KR^3 \ln 2R}{6} - \frac{2KR^3}{9} \therefore$$

Adding to the term already done before.

$$\frac{0.25K \ln(R+d) \left(-\frac{R^3}{3} - \frac{d^3}{3} \right) + 0.25K \left(-\frac{5R^3}{18} + \frac{R^2d}{3} - \frac{Rd^2}{6} + \frac{d^3}{9} \right)}{6} + \frac{KR^3 \ln 2R}{6}$$

Term 4

$$\int_R^d 0.25K d^2 \ln(d-R) dd \quad \begin{array}{l} u = d - R \quad du = dd \quad dd = du \\ d = u + R \end{array}$$

$$0.25K \int_R^d (u+R)^2 \ln u \, du \therefore 0.25K \int_R^d (u^2 + 2uR + R^2) \ln u \, du \therefore$$

$$0.25K \int_R^d (u^2 \ln u + 2uR \ln u + R^2 \ln u) \, du \therefore$$

formula of integral $\int x^m \ln x \, dx = x^{m+1} \left(\frac{\ln x}{m+1} - \frac{1}{(m+1)^2} \right)$

$$0.25K \left(u^3 \left(\frac{\ln u}{3} - \frac{1}{9} \right) + 2Ru^2 \left(\frac{\ln u}{2} - \frac{1}{4} \right) + R^2(u \ln u - u) \right) \therefore$$

$d-R$

$$\left| \frac{0.25K u^3 \ln u}{3} - \frac{0.25K u^3}{9} + 0.25KR u^2 \ln u - 0.125KR u^2 \right|_0^{d-R}$$

$$+ 0.25KR^2 u \ln u - 0.25KR^2 u \therefore$$

$$u = d-R \quad p/ \quad d=R \quad u=R-R \quad u=0$$

$$u = d-R \quad p/ \quad d=d \quad u=d-R$$

$$\frac{0.25K(d-R)^3 \ln(d-R)}{3} - \frac{0.25K(d-R)^3}{9} + 0.25KR(d-R)^2 \ln(d-R)$$

$$- 0.125KR(d-R)^2 + 0.25KR^2(d-R) \ln(d-R) - 0.25KR^2(d-R) \therefore$$

Resolving the polynomials $(R-d)^3$, $(R-d)^2$ and resolving this long algebraic expression results in:

$$0.25K \ln(d-R) \left(\frac{d^3}{3} - d^2R + dR^2 - \frac{R^3}{3} + Rd^2 - 2dR^2 + R^3 + R^2d - R^3 \right)$$

$$- 0.25K \left(\frac{d^3}{9} - \frac{d^2R}{3} + \frac{dR^2}{3} - \frac{R^3}{9} + \frac{Rd^2}{2} - \frac{dR^2}{2} + \frac{R^3}{2} + R^2d - R^3 \right)$$

$$0.25K \ln(d-R) \left(\frac{d^3}{3} - \frac{R^3}{3} \right) - 0.25K \left(\frac{d^3}{9} + \frac{d^2R}{6} + \frac{dR^2}{3} - \frac{11R^3}{18} \right)$$

Term 5

$$\int_R^d 0.5KR d \, dd$$

$$\frac{0.5KR d^2}{2}$$

$$\int_R^d 0.25KR d^2$$

$$0.25KR d^2 - 0.25KR R^2$$

$$\underline{0.25K(Rd^2 - R^3)}$$

Adding the 5 terms

$$0.25K \ln(d+R) \left(R^3 + R^2d - \frac{R^3}{3} - \frac{d^3}{3} \right)$$

$$0.25K \ln(d-R) \left(-R^2d + R^3 + \frac{d^3}{3} - \frac{R^3}{3} \right)$$

$$+ 0.5K \ln 2R \left(-R^3 + \frac{R^3}{3} \right)$$

$$+ 0.25K \left(-R^2d + R^3 + R^2d - R^3 - \frac{5R^3}{18} + \frac{R^2d}{3} - \frac{Rd^2}{6} + \frac{d^3}{9} - \frac{d^3}{9} - \frac{d^2R}{6} - \frac{dR^2}{3} \right. \\ \left. + \frac{4R^3}{18} + Rd^2 - R^3 \right) \therefore$$

$$0.25K \ln(d+R) \left(\frac{2R^3}{3} + R^2d - \frac{d^3}{3} \right) + 0.25K \ln(d-R) \left(\frac{2R^3}{3} - R^2d + \frac{d^3}{3} \right)$$

$$- \frac{KR^3}{3} \ln 2R + 0.25K \left(-\frac{2R^3}{3} + \frac{2Rd^2}{3} \right) \therefore$$

$$0.25K \ln(d+R) \left(2R^3 + 3R^2d - d^3 \right) + 0.25K \ln(d-R) \left(2R^3 - 3R^2d + d^3 \right)$$

$$- \frac{0.25K 4R^3}{3} \ln 2R + \frac{0.25K}{3} \left(-2R^3 + 2Rd^2 \right) \therefore$$

$$K = \frac{\Delta 2\pi}{V_c} \therefore \frac{0.25K}{3} = \frac{0.25 \Delta 2\pi}{3 V_c} \therefore \frac{0.25K}{3} = \frac{0.5\pi \Delta}{3 V_c}$$

$$E = \frac{\pi \Delta}{3V_c} \left[0.5 \ln(d+R) (2R^3 + 3R^2d + d^3) + 0.5 \ln(d-R) (2R^3 - 3R^2d + d^3) - 2R^3 \ln 2R - R^3 + Rd^2 \right]$$

Formula of the part disk outside

Now let's test the integration using the excel spreadsheet. Using the formula we calculate the value of E to $\frac{\pi \Delta}{3V_c} = 1$ $R=50$ $d=65$.

$$E = 1 \left[0.5 \ln(65+50) (2 \times 50^3 + 3 \times 50^2 \times 65 - 65^3) + 0.5 \ln(65-50) (2 \times 50^3 - 3 \times 50^2 \times 65 + 65^3) - 2 \times 50^3 \ln 2 \times 50 - 50^3 + 50 \times 65^2 \right]$$

$$E = 83380.86482$$

Now let's use the excel spreadsheet.

$$E_0 = \frac{K(R^2 - d^2)}{4} ((\ln(R+d) - \ln(d-R)) + 0.5KRd) dd$$

$$\text{para } \frac{\pi \Delta}{3V_c} = 1 \quad \frac{\pi \Delta}{V_c} = 3 \quad \frac{\Delta 2\pi}{V_c} = 6 \quad K=6$$

$$R=50 \quad d=65 \quad dd=0.1$$

This corresponds to 150 small pieces to be added (which is the integral concept).

$$E_0 = \frac{6(50^2 - d^2)}{4} ((\ln(50+d) - \ln(d-50)) + 0.5 \times 6 \times 50 \times d) 0.1$$

$$E_0 = 0.15 (2500 - d^2) ((\ln(50+d) - \ln(d-50)) + 15d)$$

The d values is the midpoint of each segment.

$$E = d(50.05) + d(50.15) + \dots + d(64.95)$$

$$E = 745.0460977 + 737.5945244 + \dots + 448.4434115$$

$$E = 83380.45530$$

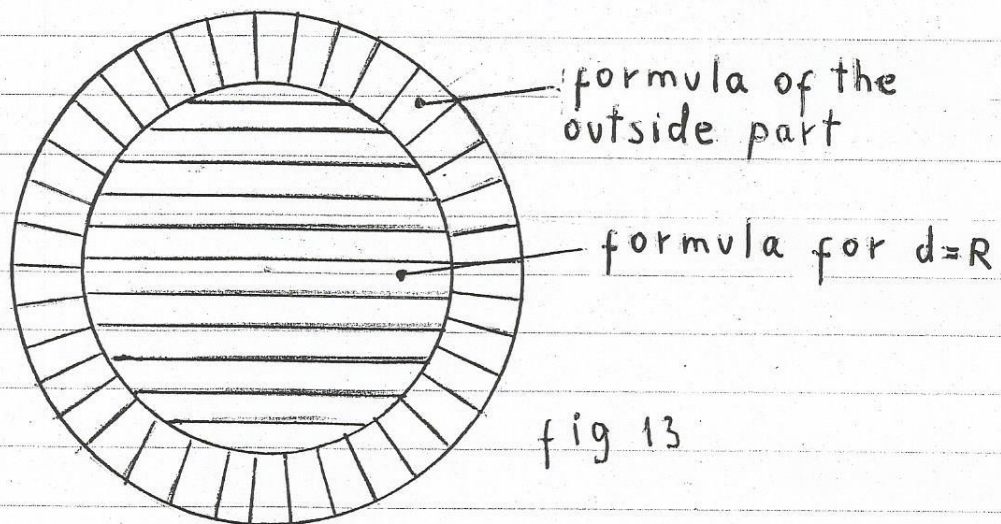
The value is very similar to the integrated formula, so it is correct.

Below is the excel spreadsheet.

d	E ₀	d	E ₀	d	E ₀	d	E ₀
50.05	745.0460977	53.85	610.1452619	57.65	538.1187938	61.45	486.1677893
50.15	737.5945244	53.95	607.8374984	57.75	536.5521829	61.55	484.9765404
50.25	731.2164513	54.05	605.5593459	57.85	534.9984232	61.65	483.7928200
50.35	725.4389728	54.15	603.3099372	57.95	533.4573023	61.75	482.6165386
50.45	720.0805943	54.25	601.0884485	58.05	531.9286139	61.85	481.4476083
50.55	715.0436047	54.35	598.8940962	58.15	530.4121570	61.95	480.2859430
50.65	710.2667232	54.45	596.7261344	58.25	528.9077358	62.05	479.1314580
50.75	705.7078871	54.55	594.5838518	58.35	527.4151598	62.15	477.9840703
50.85	701.3364312	54.65	592.4665699	58.45	525.9342434	62.25	476.8436984
50.95	697.1290107	54.75	590.3736405	58.55	524.4648056	62.35	475.7102622
51.05	693.0672631	54.85	588.3044440	58.65	523.0066701	62.45	474.5836831
51.15	689.1363711	54.95	586.2583873	58.75	521.5596648	62.55	473.4638838
51.25	685.3241293	55.05	584.2349026	58.85	520.1236220	62.65	472.3507886
51.35	681.6203124	55.15	582.2334455	58.95	518.6983781	62.75	471.2443228
51.45	678.0162316	55.25	580.2534936	59.05	517.2837734	62.85	470.1444134
51.55	674.5044146	55.35	578.2945457	59.15	515.8796519	62.95	469.0509882
51.65	671.0783683	55.45	576.3561200	59.25	514.4858615	63.05	467.9639766
51.75	667.7323992	55.55	574.4377533	59.35	513.1022536	63.15	466.8833091
51.85	664.4614758	55.65	572.5389999	59.45	511.7286830	63.25	465.8089173
51.95	661.2611201	55.75	570.6594309	59.55	510.3650077	63.35	464.7407341
52.05	658.1273219	55.85	568.7986329	59.65	509.0110893	63.45	463.6786933
52.15	655.0564699	55.95	566.9562074	59.75	507.6667921	63.55	462.6227301
52.25	652.0452954	56.05	565.1317702	59.85	506.3319837	63.65	461.5727805
52.35	649.0908267	56.15	563.3249503	59.95	505.0065346	63.75	460.5287817
52.45	646.1903501	56.25	561.5353896	60.05	503.6903179	63.85	459.4906719
52.55	643.3413789	56.35	559.7627419	60.15	502.3832098	63.95	458.4583902
52.65	640.5416259	56.45	558.0066728	60.25	501.0850889	64.05	457.4318768
52.75	637.7889807	56.55	556.2668587	60.35	499.7958364	64.15	456.4110728

52.85	635.0814904	56.65	554.5429865	60.45	498.5153362	64.25	455.3959202
52.95	632.4173428	56.75	552.8347532	60.55	497.2434745	64.35	454.3863621
53.05	629.7948517	56.85	551.1418652	60.65	495.9801398	64.45	453.3823422
53.15	627.2124450	56.95	549.4640380	60.75	494.7252230	64.55	452.3838053
53.25	624.6686529	57.05	547.8009959	60.85	493.4786172	64.65	451.3906969
53.35	622.1620990	57.15	546.1524715	60.95	492.2402175	64.75	450.4029636
53.45	619.6914915	57.25	544.5182052	61.05	491.0099214	64.85	449.4205524
53.55	617.2556160	57.35	542.8979454	61.15	489.7876282	64.95	448.4434115
53.65	614.8533286	57.45	541.2914474	61.25	488.5732393	Total E = 83380.4553000	
53.75	612.4835504	57.55	539.6984738	61.35	487.3666579		

Now let's add up the two formulas for $d = R$ and the outside part (figure 13).



for $d = R$

$$E = \frac{\pi \Delta}{3V_c} \left[0.5 \ln(R+d)(2R^3 + 3R^2d - d^3) + 0.5 \ln(R-d)(2R^3 - 3R^2d + d^3) - 2R^3 \ln R + Rd^2 \right]$$

$$E = \frac{\pi \Delta}{3V_c} \left[0.5 \ln(R+R)(2R^3 + 3R^2R - R^3) + 0.5 \ln(R-R)(2R^3 - 3R^2R + R^3) - 2R^3 \ln R + RR^2 \right]$$

$$E = \frac{\pi \Delta}{3V_c} \left[0.5 \ln 2R (4R^3) + 0.5 \ln(0) \times (0) - 2R^3 \ln R + R^3 \right]$$

$$E = \frac{\pi \Delta}{3V_c} \left[2R^3 \ln 2R - 2R^3 \ln R + R^3 \right]$$

Adding the two formulas.

$$E = \frac{\pi \Delta}{3V_c} \left[0.5 \ln(d+R)(2R^3 + 3R^2d - d^3) + 0.5 \ln(d-R)(2R^3 - 3R^2d + d^3) - 2R^3 \ln R + Rd^2 \right] \quad \text{disk formula for } d > R$$

The formula for $d > R$ is practically the same formula for the internal disk ($d \leq R$) differing only in item $\ln(R-d)$ and $\ln(d-R)$. If to make the formula of the internal disk $\ln(|R-d|)$, it becomes a unique formula.

$$E = \frac{\pi \Delta}{3V_c} \left[0.5 \ln(R+d)(2R^3 + 3R^2d - d^3) + 0.5 \ln(|R-d|)(2R^3 - 3R^2d + d^3) - 2R^3 \ln R + Rd^2 \right] \quad \text{disk formula}$$

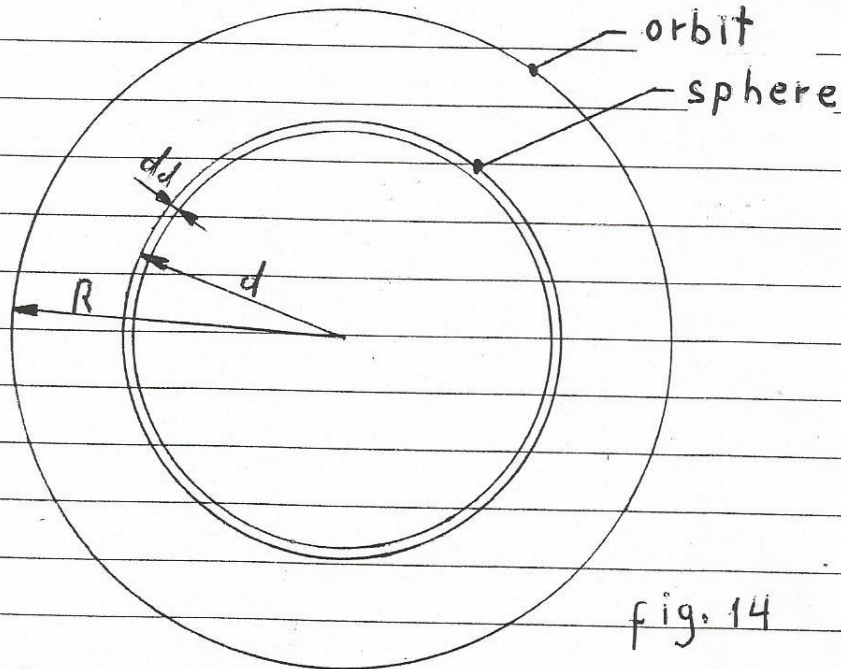
Sphere formula

The formula of the sphere will be deduced similar to the disk formula. Let us consider a hollow sphere of a very small thickness (dd), all points on it will have the distance d from the center (figure 14). After integrating E_0 in function of d , we will have the sum of all small hollow spheres, that is, the total energy of the sphere. The sphere will have the constant power of the volume, $S = P / \text{Vol}$ S in W / m^3 .

The formula from the point out of center:

$$E = K_0 \left(\frac{R^2 - d^2}{4d} \right) (\ln(R+d) - \ln(R-d)) + 0.5 K_0 R \quad K_0 = \frac{P}{V_c}$$

$$E = \frac{P}{V_c} \left(\left(\frac{R^2 - d^2}{4d} \right) (\ln(R+d) - \ln(R-d)) + 0.5 R \right)$$



$$P = S \times \text{Vol} \quad P = S \times 4\pi d^2 \times dd$$

$$E_0 = \frac{S \times 4\pi d^2}{V_c} \left(\frac{R^2 - d^2}{4d} (\ln(R+d) - \ln(R-d)) + 0.5R \right)$$

$$K = \frac{S \times 4\pi}{V_c}$$

$$E_0 = K d^2 \left(\frac{R^2 - d^2}{4d} (\ln(R+d) - \ln(R-d)) + 0.5R \right) dd$$

$$E_0 = \frac{K d (R^2 - d^2)}{4} ((\ln(R+d) - \ln(R-d)) + 0.5R K d^2) dd$$

$$E_0 = (0.25 K d R^2 - 0.25 K d^3) ((\ln(R+d) - \ln(R-d)) + 0.5R K d^2) dd$$

$$E_0 = \overset{(1)}{0.25 K d R^2 \ln(R+d)} dd - \overset{(2)}{0.25 K d R^2 \ln(R-d)} dd$$

$$- \overset{(3)}{0.25 K d^3 \ln(R+d)} dd + \overset{(4)}{0.25 K d^3 \ln(R-d)} + \overset{(5)}{0.5 R K d^2} dd$$

Now let's do the integration of E_0 in function of d , it is a very laborious and difficult integral, so we need to do it step by step, integrating the 5 terms separately.

Term 1

$$\int 0.25 K d R^2 \ln(R+d) dd \quad \begin{array}{l} u = R+d \quad du = dd \\ d = u - R \end{array}$$

$$0.25 K R^2 \int d \ln(R+d) dd \therefore 0.25 K R^2 \int (u-R) \ln u du$$

$$0.25KR^2 \int u \ln u - R \ln u \, du$$

$$0.25KR^2 \left(u^2 \left(\frac{\ln u}{2} - \frac{1}{4} \right) - R(u \ln u - u) \right)$$

$$0.25KR^2 \left(\frac{u^2 \ln u}{2} - \frac{u^2}{4} - Ru \ln u + Ru \right)$$

$R+d$

$$\left| \begin{array}{l} 0.125KR^2 u^2 \ln u - 0.0625KR^2 u^2 - 0.25KR^3 u \ln u + 0.25KR^3 u \\ R \end{array} \right.$$

$$\begin{array}{l} u=R+d \quad p/d=0 \quad u=R+0 \quad u=R \\ p/d=d \quad u=R+d \end{array}$$

$$\begin{aligned} & 0.125KR^2(R+d)^2 \ln(R+d) - 0.0625KR^2(R+d)^2 \\ & - 0.25KR^3(R+d) \ln(R+d) + 0.25KR^3(R+d) \\ & - (0.125K \times R^2 \times R^2 \ln R - 0.0625K \times R^2 \times R^2 - 0.25KR^3 \times R \ln R \\ & + 0.25KR^3 \times R) \therefore \end{aligned}$$

$$\begin{aligned} & (0.125KR^4 + 0.25KR^3d + 0.125KR^2d^2) \ln(R+d) - 0.0625KR^4 \\ & - 0.125KR^3d - 0.0625KR^2d^2 + (-0.25KR^4 - 0.25KR^3d) \ln(R+d) \\ & + 0.25KR^4 + 0.25KR^3d - 0.125KR^4 \ln R + 0.0625KR^4 + 0.25KR^4 \ln R \\ & - 0.25KR^4 \end{aligned}$$

$$\begin{aligned} & \ln(R+d) (-0.125KR^4 + 0.125KR^2d^2) + 0.125KR^4 \ln R \\ & + 0.125KR^3d - 0.0625KR^2d^2 \end{aligned}$$

Term 2

$$\int -0.25 K d R^2 \ln(R-d) dd \quad \begin{array}{l} u=R-d \quad du=-dd \quad dd=-du \\ -d=u-R \quad d=R-u \end{array}$$

$$-0.25 K R^2 \int d \ln u \cdot -du \therefore 0.25 K R^2 \int (R-u) \cdot \ln u \cdot du$$

$$0.25 K R^2 \int R \ln u - u \ln u du$$

$$0.25 K R^2 \left(R \left(u \ln u - u \right) - u^2 \left(\frac{\ln u}{2} - \frac{1}{4} \right) \right)$$

$$0.25 K R^2 \left(R u \ln u - R u - \frac{u^2 \ln u}{2} + \frac{u^2}{4} \right)$$

$$\left. \begin{array}{l} R-d \\ R \end{array} \right| 0.25 K R^3 u \ln u - 0.25 K R^3 u - 0.125 K R^2 u^2 \ln u + 0.0625 K R^2 u^2$$

$$u=R-d \quad p/d=0 \quad u=R-0 \quad u=R \quad p/d=d \quad u=R-d$$

$$0.25 K R^3 (R-d) \ln(R-d) - 0.25 K R^3 (R-d) - 0.125 K R^2 (R-d)^2 \ln(R-d) + 0.0625 K R^2 (R-d)^2 - (0.25 K R^3 R \ln R - 0.25 K R^3 R - 0.125 K R^2 R^2 \ln R + 0.0625 K R^2 R^2) \therefore$$

$$0.25 K R^4 \ln(R-d) - 0.25 K R^3 d \ln(R-d) - 0.25 K R^4 + 0.25 K R^3 d - 0.125 K R^4 \ln(R-d) + 0.25 K R^3 d \ln(R-d) - 0.125 K R^2 d^2 \ln(R-d) + 0.0625 K R^4 - 0.125 K R^3 d + 0.0625 K R^2 d^2 - 0.25 K R^4 \ln R + 0.25 K R^4 + 0.125 K R^4 \ln R - 0.0625 K R^4 \therefore$$

$$0.125 K R^4 \ln(R-d) + 0.125 K R^3 d - 0.125 K R^2 d^2 \ln(R-d) + 0.0625 K R^2 d^2 - 0.125 K R^4 \ln R$$

$$\ln(R-d)(0.125KR^4 - 0.125KR^2d^2) - 0.125KR^4 \ln R + 0.125KR^3d + 0.0625KR^2d^2$$

Term 3

$$\int -0.25Kd^3 \ln(R+d) dd \quad \begin{array}{l} u = R+d \quad du = dd \\ d = u - R \end{array}$$

$$-0.25K \int (u-R)^3 \ln u du$$

$$-0.25K \int (u^3 - 3u^2R + 3uR^2 - R^3) \ln u du$$

$$-0.25K \int (u^3 \ln u - 3u^2R \ln u + 3uR^2 \ln u - R^3 \ln u) du$$

$$-0.25K \left(u^4 \left(\frac{\ln u}{4} - \frac{1}{16} \right) - 3Ru^3 \left(\frac{\ln u}{3} - \frac{1}{9} \right) + 3R^2u^2 \left(\frac{\ln u}{2} - \frac{1}{4} \right) \right.$$

$$\left. - R^3(u \ln u - u) \right) \therefore$$

$$-0.25K \left| \begin{array}{l} R+d \\ R \end{array} \right. 0.25u^4 \ln u - 0.0625u^4 - Ru^3 \ln u + \frac{Ru^3}{3} + 1.5R^2u^2 \ln u$$

$$- 0.75R^2u^2 - R^3u \ln u + R^3u$$

$$u = R+d \quad p/d=0 \quad u = R+0 \quad u = R \quad p/d=d \quad u = R+d$$

$$-0.25K \left(0.25(R+d)^4 \ln(R+d) - 0.0625(R+d)^4 - R(R+d)^3 \ln(R+d) + \frac{R(R+d)^3}{3} + 1.5R^2(R+d)^2 \ln(R+d) - 0.75R^2(R+d)^2 - R^3(R+d) \ln(R+d) \right)$$

$$+ R^3(R+d) - \left(0.25R^4 \ln R - 0.0625R^4 - RR^3 \ln R + \frac{RR^3}{3} + 1.5R^2R^2 \ln R - 0.75R^2R^2 - R^3R \ln R + R^3R \right) \therefore$$

$$\begin{aligned} & -0.25K(0.25R^4 \ln(R+d) + R^3d \ln(R+d) + 1.5R^2d^2 \ln(R+d) + Rd^3 \ln(R+d) \\ & + 0.25d^4 \ln(R+d) - 0.0625R^4 - 0.25R^3d - 0.375R^2d^2 - 0.25Rd^3 \\ & - 0.0625d^4 - R^4 \ln(R+d) - 3R^3d \ln(R+d) - 3R^2d^2 \ln(R+d) - Rd^3 \ln(R+d) \\ & + \frac{R^4}{3} + R^3d + R^2d^2 + \frac{Rd^3}{3} + 1.5R^4 \ln(R+d) + 3R^3d \ln(R+d) \\ & + 1.5R^2d^2 \ln(R+d) - 0.75R^4 - 1.5R^3d - 0.75R^2d^2 - R^4 \ln(R+d) \\ & - R^3d \ln(R+d) + R^4 + R^3d - 0.25R^4 \ln R + 0.0625R^4 + R^4 \ln R \\ & - \frac{R^4}{3} - 1.5R^4 \ln R + 0.75R^4 + R^4 \ln R - R^4) \therefore \end{aligned}$$

$$\begin{aligned} & -0.25K(-0.25R^4 \ln(R+d) + 0.25d^4 \ln(R+d) + 0.25R^3d \\ & - 0.125R^2d^2 + \frac{Rd^3}{12} - 0.0625d^4 + 0.25R^4 \ln R) \therefore \end{aligned}$$

$$\begin{aligned} & -0.25K(\ln(R+d)(-0.25R^4 + 0.25d^4) + 0.25R^4 \ln R + 0.25R^3d \\ & - 0.125R^2d^2 + \frac{Rd^3}{12} - 0.0625d^4) \end{aligned}$$

$$\begin{aligned} & \ln(R+d)(0.0625KR^4 - 0.0625Kd^4) - 0.0625KR^4 \ln R \\ & - 0.0625KR^3d + 0.03125KR^2d^2 - \frac{KRd^3}{48} + \frac{Kd^4}{64} \end{aligned}$$

Term 4

$$\int 0.25 K d^3 \ln(R-d) dd \quad \begin{array}{l} u=R-d \quad du=-dd \quad dd=-du \\ -d=u-R \quad d=R-u \end{array}$$

$$0.25 K \int (R-u)^3 \ln u \cdot -du$$

$$-0.25 K \int (R^3 - 3R^2u + 3Ru^2 - u^3) \ln u du$$

$$-0.25 K \int (R^3 \ln u - 3R^2u \ln u + 3Ru^2 \ln u - u^3 \ln u) du$$

$$0.25 K \int (u^3 \ln u - 3u^2 R \ln u + 3u R^2 \ln u - R^3 \ln u) du$$

$$0.25 K \left(u^4 \left(\frac{\ln u}{4} - \frac{1}{16} \right) - 3R u^3 \left(\frac{\ln u}{3} - \frac{1}{9} \right) + 3R^2 u^2 \left(\frac{\ln u}{2} - \frac{1}{4} \right) - R^3 (u \ln u - u) \right)$$

$$0.25 K \left| \begin{array}{l} R-d \\ R \end{array} \right. \begin{array}{l} 0.25 u^4 \ln u - 0.0625 u^4 - R u^3 \ln u + \frac{R u^3}{3} + 1.5 R^2 u^2 \ln u \\ - 0.75 R^2 u^2 - R^3 u \ln u + R^3 u \end{array}$$

$$u=R-d \quad p/d=0 \quad u=R \quad p/d=d \quad u=R-d$$

$$\begin{aligned} & 0.25 K \left(0.25 (R-d)^4 \ln(R-d) - 0.0625 (R-d)^4 - R (R-d)^3 \ln(R-d) \right. \\ & \left. \frac{R (R-d)^3}{3} + 1.5 R^2 (R-d)^2 \ln(R-d) - 0.75 R^2 (R-d)^2 - R^3 (R-d) \ln(R-d) \right. \\ & \left. + R^3 (R-d) - \left(0.25 R^4 \ln R - 0.0625 R^4 - R R^3 \ln R + \frac{R R^3}{3} \right. \right. \\ & \left. \left. + 1.5 R^2 R^2 \ln R - 0.75 R^2 R^2 - R^3 R \ln R + R^3 R \right) \right) \therefore \end{aligned}$$

$$\begin{aligned}
& 0.25K(0.25R^4 \ln(R-d) - R^3 d \ln(R-d) + 1.5R^2 d^2 \ln(R-d) \\
& - R d^3 \ln(R-d) + 0.25d^4 \ln(R-d) - 0.0625R^4 + 0.25R^3 d \\
& - 0.375R^2 d^2 + 0.25R d^3 - 0.0625d^4 - R^4 \ln(R-d) + 3R^3 d \ln(R-d) \\
& - 3R^2 d^2 \ln(R-d) + R d^3 \ln(R-d) + \frac{R^4}{3} - R^3 d + R^2 d^2 - \frac{R d^3}{3} \\
& + 1.5R^4 \ln(R-d) - 3R^3 d \ln(R-d) + 1.5R^2 d^2 \ln(R-d) - 0.75R^4 \\
& + 1.5R^3 d - 0.75R^2 d^2 - R^4 \ln(R-d) + R^3 d \ln(R-d) + R^4 \\
& - R^3 d - 0.25R^4 \ln R + 0.0625R^4 + R^4 \ln R - \frac{R^4}{3} - 1.5R^4 \ln R \\
& + 0.75R^4 + R^4 \ln R - R^4) \therefore
\end{aligned}$$

$$\begin{aligned}
& 0.25K(-0.25R^4 \ln(R-d) + 0.25d^4 \ln(R-d) - 0.25R^3 d \\
& - 0.125R^2 d^2 - \frac{R d^3}{12} - 0.0625d^4 + 0.25R^4 \ln R)
\end{aligned}$$

$$\begin{aligned}
& \ln(R-d)(-0.0625KR^4 + 0.0625Kd^4) + 0.0625KR^4 \ln R \\
& - 0.0625KR^3 d - 0.03125KR^2 d^2 - \frac{KR d^3}{48} - \frac{Kd^4}{64}
\end{aligned}$$

Term 5

$$\int 0.5KR d^2 dd$$

$$0.5KR \int d^2 dd \therefore \frac{0.5KR d^3}{3}$$

Adding the 5 terms

$$\begin{aligned} & \ln(R+d)(-0.0625KR^4 + 0.125KR^2d^2 - 0.0625Kd^4) \\ & + \ln(R-d)(0.0625KR^4 - 0.125KR^2d^2 + 0.0625Kd^4) \\ & + 0.125KR^3d + 0.125KRd^3 \therefore \end{aligned}$$

$$\begin{aligned} E = 0.125K & \left[\ln(R+d)(-0.5R^4 + R^2d^2 - 0.5d^4) \right. \\ & \left. + \ln(R-d)(0.5R^4 - R^2d^2 + 0.5d^4) + R^3d + Rd^3 \right] \therefore \end{aligned}$$

$$K = \frac{5.4\pi}{V_c}$$

$$\begin{aligned} E = \frac{\pi S}{2V_c} & \left[0.5 \ln(R+d)(-R^4 + 2R^2d^2 - d^4) \right. \\ & \left. + 0.5 \ln(R-d)(R^4 - 2R^2d^2 + d^4) + R^3d + Rd^3 \right] \end{aligned}$$

Formula of the internal sphere $d \leq R$

Now let's test the integration using the excel spreadsheet. Using the formula we calculate the value of E to $\frac{\pi S}{2V_c} = 1$ $R=50$ $d=30$.

$$\begin{aligned} E &= 1 \left[0.5 \ln(50+30)(-50^4 + 2 \times 50^2 \times 30^2 - 30^4) \right. \\ & \left. + 0.5 \ln(50-30)(50^4 - 2 \times 50^2 \times 30^2 + 30^4) + 50^3 \times 30 + 50 \times 30^3 \right] \\ E &= 3325543.218 \end{aligned}$$

Now let's use the excel spreadsheet.

$$E_0 = \frac{Kd(R^2 - d^2)((\ln(R+d) - \ln(R-d)) + 0.5Rkd^2)}{4} dd$$

$$\text{para } \frac{\pi S}{2V_c} = 1 \quad \frac{\pi S}{V_c} = 2 \quad \frac{54\pi}{V_c} = 8 \quad K=8$$

$$R=50 \quad d=30 \quad dd=0.2$$

This corresponds to 150 small pieces to be added (which is the integral concept).

$$E_0 = \frac{8 \times d(50^2 - d^2)((\ln(50+d) - \ln(50-d)) + 0.5 \times 50 \times 8 \times d^2)}{4} 0.2$$

$$E_0 = 0.4d(2500 - d^2)(\ln(50+d) - \ln(50-d)) + 40d^2$$

The d values is the midpoint of each segment.

$$E = d(29.9) + d(29.7) + \dots + d(0.1)$$

$$E = 62268.02010 + 61570.86219 + \dots + 0.8$$

$$E = 3325514.142$$

The value is very similar to the integrated formula, so it is correct.

Below is the excel spreadsheet.

d	E ₀	d	E ₀	d	E ₀	d	E ₀
29.9	62268.02010	22.3	37030.33978	14.7	16780.17563	7.1	4005.58394
29.7	61570.86219	22.1	36419.54979	14.5	16340.24839	6.9	3784.52889
29.5	60874.46006	21.9	35812.36848	14.3	15905.59606	6.7	3569.62768
29.3	60178.91078	21.7	35208.85226	14.1	15476.25085	6.5	3360.89451
29.1	59484.30994	21.5	34609.05675	13.9	15052.24445	6.3	3158.34312
28.9	58790.75166	21.3	34013.03683	13.7	14633.60801	6.1	2961.98686
28.7	58098.32868	21.1	33420.84660	13.5	14220.37215	5.9	2771.83860

28.5	57407.13233	20.9	32832.53943	13.3	13812.56700	5.7	2587.91084
28.3	56717.25259	20.7	32248.16794	13.1	13410.22214	5.5	2410.21559
28.1	56028.77810	20.5	31667.78404	12.9	13013.36669	5.3	2238.76448
27.9	55341.79619	20.3	31091.43890	12.7	12622.02920	5.1	2073.56870
27.7	54656.39294	20.1	30519.18298	12.5	12236.23778	4.9	1914.63902
27.5	53972.65314	19.9	29951.06604	12.3	11856.01999	4.7	1761.98577
27.3	53290.66038	19.7	29387.13717	12.1	11481.40291	4.5	1615.61889
27.1	52610.49701	19.5	28827.44472	11.9	11112.41314	4.3	1475.54787
26.9	51932.24424	19.3	28272.03641	11.7	10749.07677	4.1	1341.78179
26.7	51255.98206	19.1	27720.95926	11.5	10391.41941	3.9	1214.32932
26.5	50581.78938	18.9	27174.25964	11.3	10039.46619	3.7	1093.19870
26.3	49909.74393	18.7	26631.98326	11.1	9693.24174	3.5	978.39776
26.1	49239.92238	18.5	26094.17516	10.9	9352.77023	3.3	869.93391
25.9	48572.40031	18.3	25560.87977	10.7	9018.07534	3.1	767.81415
25.7	47907.25222	18.1	25032.14085	10.5	8689.18029	2.9	672.04506
25.5	47244.55157	17.9	24508.00156	10.3	8366.10782	2.7	582.63280
25.3	46584.37081	17.7	23988.50441	10.1	8048.88020	2.5	499.58312
25.1	45926.78135	17.5	23473.69130	9.9	7737.51923	2.3	422.90138
24.9	45271.85363	17.3	22963.60353	9.7	7432.04625	2.1	352.59248
24.7	44619.65710	17.1	22458.28178	9.5	7132.48215	1.9	288.66095
24.5	43970.26024	16.9	21957.76613	9.3	6838.84734	1.7	231.11089
24.3	43323.73059	16.7	21462.09607	9.1	6551.16178	1.5	179.94599
24.1	42680.13477	16.5	20971.31050	8.9	6269.44499	1.3	135.16953
23.9	42039.53845	16.3	20485.44773	8.7	5993.71601	1.1	96.78438
23.7	41402.00642	16.1	20004.54551	8.5	5723.99345	0.9	64.79300
23.5	40767.60257	15.9	19528.64098	8.3	5460.29546	0.7	39.19744
23.3	40136.38990	15.7	19057.77076	8.1	5202.63974	0.5	19.99933
23.1	39508.43056	15.5	18591.97086	7.9	4951.04356	0.3	7.19991
22.9	38883.78585	15.3	18131.27676	7.7	4705.52374	0.1	0.80000
22.7	38262.51620	15.1	17675.72336	7.5	4466.09664	Total E= 3325514.14200	
22.5	37644.68125	14.9	17225.34505	7.3	4232.77821		

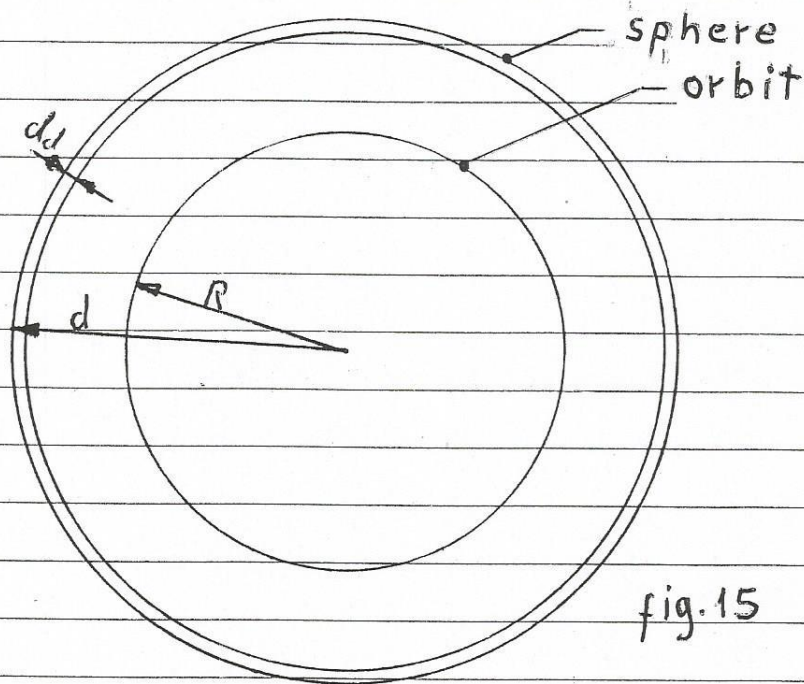
Just as it occurred with the disk, we will do with the sphere, we must check the situation in which the orbit is inside the sphere. We deduce the formula for part of the sphere which is outside the orbit and then will add to the part of the sphere is equal to the orbit. The steps will be similar to the internal sphere (figure 15).

Below is the formula of the external point:

$$E = K_0 \left(\frac{R^2 - d^2}{4d} \right) (\ln(d+R) - \ln(d-R)) + 0.5 K_0 R$$

onde $K_0 = \frac{P}{V_c}$

$$E = \frac{P}{V_c} \left(\left(\frac{R^2 - d^2}{4d} \right) (\ln(d+R) - \ln(d-R)) + 0.5R \right)$$



$$P = S \cdot V_0 \quad P = S \cdot 4\pi d^2 \cdot dd$$

$$E_0 = \frac{S \cdot 4\pi d^2 \cdot dd}{V_c} \left(\left(\frac{R^2 - d^2}{4d} \right) (\ln(d+R) - \ln(d-R)) + 0.5R \right)$$

$$K = \frac{S \cdot 4\pi}{V_c}$$

$$E_0 = K d^2 \left(\left(\frac{R^2 - d^2}{4d} \right) (\ln(d+R) - \ln(d-R)) + 0.5R \right) dd$$

$$E_0 = \frac{Kd(R^2 - d^2)}{4} ((\ln(d+R) - \ln(d-R)) + 0.5RKd^2) dd$$

$$E_0 = (0.25KdR^2 - 0.25Kd^3) ((\ln(d+R) - \ln(d-R)) + 0.5RKd^2) dd$$

$$E_0 = \overset{(1)}{0.25KdR^2 \ln(d+R) dd} - \overset{(2)}{0.25KdR^2 \ln(d-R) dd}$$

$$- \overset{(3)}{0.25Kd^3 \ln(d+R) dd} + \overset{(4)}{0.25Kd^3 \ln(d-R) dd} + \overset{(5)}{0.5RKd^2 dd}$$

The terms 1, 3 and 5 are identical, the integral will be the same, the integration interval is different from R to d.

Term 1

Interval from R to d, integration already done

$$\int_{2R}^{R+d} 0.125KR^2 u^2 \ln u - 0.0625KR^2 u^2 - 0.25KR^3 u \ln u + 0.25KR^3 u$$

$$\begin{aligned} u &= R+d & p/d &= R & u &= R+R & u &= 2R \\ p/d &= d & u &= R+d \end{aligned}$$

$$\begin{aligned} &0.125KR^2(R+d)^2 \ln(R+d) - 0.0625KR^2(R+d)^2 - 0.25KR^3(R+d) \ln(R+d) \\ &+ 0.25KR^3(R+d) - (0.125KR^2 4R^2 \ln 2R - 0.0625KR^2 4R^2 \\ &- 0.25KR^3 2R \ln 2R + 0.25KR^3 2R) \therefore \end{aligned}$$

$$\begin{aligned} &(0.125KR^4 + 0.25KR^3d + 0.125KR^2d^2) \ln(R+d) - 0.0625KR^4 \\ &- 0.125KR^3d - 0.0625KR^2d^2 + (-0.25KR^4 - 0.25KR^3d) \ln(R+d) \\ &+ 0.25KR^4 + 0.25KR^3d - 0.5KR^4 \ln 2R + 0.25KR^4 + 0.5KR^4 \ln 2R \\ &- 0.5KR^4 \therefore \end{aligned}$$

$$\ln(R+d)(-0.125KR^4 + 0.125KR^2d^2) - 0.0625KR^4 + 0.125KR^3d - 0.0625KR^2d^2$$

Term 2

$$\int -0.25KdR^2 \ln(d-R) dd \quad \begin{array}{l} u = d-R \quad du = dd \\ d = R+u \end{array}$$

$$-0.25KR^2 \int (R+u) \ln u du$$

$$-0.25KR^2 \int (R \ln u + u \ln u) du$$

$$-0.25KR^2 \left(R(u \ln u - u) + u^2 \left(\frac{\ln u}{2} - \frac{1}{4} \right) \right)$$

$$-0.25KR^2 \left(Ru \ln u - Ru + \frac{u^2 \ln u}{2} - \frac{u^2}{4} \right)$$

$$\begin{array}{l} d-R \\ \left| \begin{array}{l} -0.25KR^3 u \ln u + 0.25KR^3 u - 0.125KR^2 u^2 \ln u + 0.0625KR^2 u^2 \end{array} \right. \\ 0 \end{array}$$

$$u = d-R \quad \text{p/ } d=R \quad u=R-R \quad u=0 \quad \text{p/ } d=d \quad u=d-R$$

$$-0.25KR^3(d-R) \ln(d-R) + 0.25KR^3(d-R) - 0.125KR^2(d-R)^2 \ln(d-R) + 0.0625KR^2(d-R)^2$$

$$\begin{array}{l} -0.25KR^3d \ln(d-R) + 0.25KR^4 \ln(d-R) + 0.25KR^3d - 0.25KR^4 \\ -0.125KR^2d^2 \ln(d-R) + 0.25KR^3d \ln(d-R) - 0.125KR^4 \ln(d-R) \\ + 0.0625KR^2d^2 - 0.125KR^3d + 0.0625KR^4 \end{array}$$

$$\ln(d-R)(0.125KR^4 - 0.125KR^2d^2) - 0.1875KR^4 + 0.125KR^3d + 0.0625KR^2d^2$$

Term 3

Interval from R to d, integration already done

$$-0.25K \int_{2R}^{R+d} 0.25u^4 \ln u - 0.0625u^4 - Ru^3 \ln u + \frac{Ru^3}{3} + 1.5R^2u^2 \ln u$$

$$- 0.75R^2u^2 - R^3u \ln u + R^3u$$

$$u=R+d \quad p/d=R \quad u=R+R \quad u=2R \quad p/d=d \quad u=R+d$$

$$-0.25K(0.25(R+d)^4 \ln(R+d) - 0.0625(R+d)^4 - R(R+d)^3 \ln(R+d) + \frac{R(R+d)^3}{3} + 1.5R^2(R+d)^2 \ln(R+d) - 0.75R^2(R+d)^2 - R^3(R+d) \ln(R+d) + R^3(R+d))$$

$$- (0.25 \times 16R^4 \ln 2R - 0.0625 \times 16R^4 - R \times 8R^3 \ln 2R + \frac{R \times 8R^3}{3} + 1.5R^2 \times 4R^2 \ln 2R - 0.75R^2 \times 4R^2 - R^3 \times 2R \ln 2R + R^3 \times 2R)$$

$$-0.25K(0.25R^4 \ln(R+d) + R^3d \ln(R+d) + 1.5R^2d^2 \ln(R+d) + Rd^3 \ln(R+d) + 0.25d^4 \ln(R+d) - 0.0625R^4 - 0.25R^3d - 0.375R^2d^2 - 0.25Rd^3 - 0.0625d^4 - R^4 \ln(R+d) - 3R^3d \ln(R+d) - 3R^2d^2 \ln(R+d) - Rd^3 \ln(R+d) + \frac{R^4}{3} + R^3d + R^2d^2 + \frac{Rd^3}{3}$$

$$+ 1.5R^4 \ln(R+d) + 3R^3d \ln(R+d) + 1.5R^2d^2 \ln(R+d) - 0.75R^4 - 1.5R^3d - 0.75R^2d^2 - R^4 \ln(R+d) - R^3d \ln(R+d) + R^4 + R^3d$$

$$-4R^4 \ln 2R + R^4 + 8R^4 \ln 2R - \frac{8R^4}{3} - 6R^4 \ln 2R + 3R^4 + 2R^4 \ln 2R - 2R^4) \therefore$$

$$-0.25K (\ln(R+d)(-0.25R^4 + 0.25d^4) - \frac{7}{48}R^4 + 0.25R^3d - 0.125R^2d^2 + \frac{Rd^3}{12} - 0.0625d^4) \therefore$$

$$\ln(R+d)(0.0625KR^4 - 0.0625Kd^4) + \frac{7KR^4}{192} - 0.0625KR^3d + 0.03125KR^2d^2 - \frac{KRd^3}{48} + \frac{Kd^4}{64}$$

Term 4

$$\int 0.25Kd^3 \ln(d-R) dd \quad \begin{array}{l} u = d - R \quad du = dd \\ d = R + u \end{array}$$

$$0.25K \int (R+u)^3 \ln u du$$

$$0.25K \int (R^3 + 3R^2u + 3Ru^2 + u^3) \ln u du$$

$$0.25K \int (R^3 \ln u + 3R^2u \ln u + 3Ru^2 \ln u + u^3 \ln u) du$$

$$0.25K \int (u^3 \ln u + 3Ru^2 \ln u + 3R^2u \ln u + R^3 \ln u) du$$

$$0.25K \left(u^4 \left(\frac{\ln u}{4} - \frac{1}{16} \right) + 3Ru^3 \left(\frac{\ln u}{3} - \frac{1}{9} \right) + 3R^2u^2 \left(\frac{\ln u}{2} - \frac{1}{4} \right) + R^3(u \ln u - u) \right) \therefore$$

$$0.25K \left| \begin{array}{c} d-R \\ 0.25u^4 \ln u - 0.0625u^4 + Ru^3 \ln u - \frac{Ru^3}{3} + 1.5R^2u^2 \ln u \\ 0 \end{array} \right| - 0.75R^2u^2 + R^3u \ln u - R^3u \therefore$$

$$u = d-R \quad p/d = R \quad u = R-R \quad u = 0 \quad p/d = d \quad u = d-R$$

$$0.25K \left(0.25(d-R)^4 \ln(d-R) - 0.0625(d-R)^4 + R(d-R)^3 \ln(d-R) - R(d-R)^3 + 1.5R^2(d-R)^2 \ln(d-R) - 0.75R^2(d-R)^2 + R^3(d-R) \ln(d-R) - R^3(d-R) \right) \therefore$$

$$0.25K \left(0.25d^4 \ln(d-R) - d^3R \ln(d-R) + 1.5d^2R^2 \ln(d-R) - dR^3 \ln(d-R) + 0.25R^4 \ln(d-R) - 0.0625d^4 + 0.25d^3R - 0.375d^2R^2 + 0.25dR^3 - 0.0625R^4 + Rd^3 \ln(d-R) - 3d^2R^2 \ln(d-R) + 3dR^3 \ln(d-R) - R^4 \ln(d-R) - \frac{Rd^3}{3} + d^2R^2 - dR^3 + \frac{R^4}{3} + 1.5R^2d^2 \ln(d-R) - 3dR^3 \ln(d-R) + 1.5R^4 \ln(d-R) - 0.75R^2d^2 + 1.5dR^3 - 0.75R^4 + R^3d \ln(d-R) - R^4 \ln(d-R) - R^3d - R^4 \right) \therefore$$

$$0.25K \left(\ln(d-R) \left(0.25d^4 - 0.25R^4 \right) - 0.0625d^4 - \frac{d^3R}{12} - 0.125d^2R^2 - 0.25dR^3 + \frac{25R^4}{48} \right) \therefore$$

$$\ln(d-R)(0.0625Kd^4 - 0.0625KR^4) - \frac{Kd^4}{64} - \frac{Kd^3R}{48} - 0.03125Kd^2R^2 - 0.0625KdR^3 + \frac{25KR^4}{192}$$

Term 5

$$\int 0.5KRd^2 dd \therefore 0.5KR \int d^2 dd \therefore \frac{0.5KRd^3}{3}$$

$$\left| \frac{KRd^3}{6} \right|_R^d \therefore \frac{KRd^3}{6} - \frac{KRR^3}{6} \therefore \frac{KR}{6} (d^3 - R^3)$$

Adding the 5 terms

$$\ln(R+d)(-0.0625KR^4 + 0.125KR^2d^2 - 0.0625Kd^4)$$

$$\ln(d-R)(0.0625KR^4 + 0.125KR^2d^2 + 0.0625Kd^4) - 0.25KR^4 + 0.125KR^3d + 0.125KRd^3$$

$$E = 0.125K \left[\ln(R+d)(-0.5R^4 + R^2d^2 - 0.5d^4) + \ln(d-R)(0.5R^4 - R^2d^2 + 0.5d^4) - 2R^4 + R^3d + Rd^3 \right]$$

$$K = \frac{54\pi}{V_c}$$

$$E = \frac{\pi S}{2V_c} \left[0.5 \ln(R+d)(-R^4 + 2R^2d^2 - d^4) + 0.5 \ln(d-R)(R^4 - 2R^2d^2 + d^4) - 2R^4 + R^3d + Rd^3 \right]$$

Formula of the part sphere outside

Now let's test the integration using the excel spreadsheet. Using the formula we calculate the value of E to $\frac{\pi S}{2V_c} = 1$ $R=50$ $d=65$.

$$E = 1 \left[0.5 \ln(50+65)(-50^4 + 2 \times 50^2 \times 65^2 - 65^4) + 0.5 \ln(65-50)(50^4 - 2 \times 50^2 \times 65^2 + 65^4) - 2 \times 50^4 + 50^3 \times 65 + 50 \times 65^3 \right]$$

$$E = 6325751.608$$

Now let's use the excel spreadsheet.

$$E_0 = \frac{Kd(R^2 - d^2)}{4} ((\ln(d+R) - \ln(d-R)) + 0.5RKd^2) dd$$

$$\text{para } \frac{\pi S}{2V_c} = 1 \quad \frac{\pi S}{V_c} = 2 \quad \frac{54\pi}{V_c} = 8 \quad K=8$$

$$R=50 \quad d=65 \quad dd=0.1$$

This corresponds to 150 small pieces to be added (which is the integral concept).

$$E_0 = \frac{8d(50^2 - d^2)}{4} ((\ln(d+50) - \ln(d-50)) + 0.5 \times 50 \times 8 \times d^2) 0.1$$

$$E_0 = 0.2d(2500 - d^2)(\ln(d+50) - \ln(d-50)) + 20d^2$$

The d values is the midpoint of each segment.

$$E = d(64.95) + d(64.85) + \dots + d(50.05)$$

$$E = 38835.19943 + 38859.8971 + \dots + 49719.40959$$

$$E = 6325726.790$$

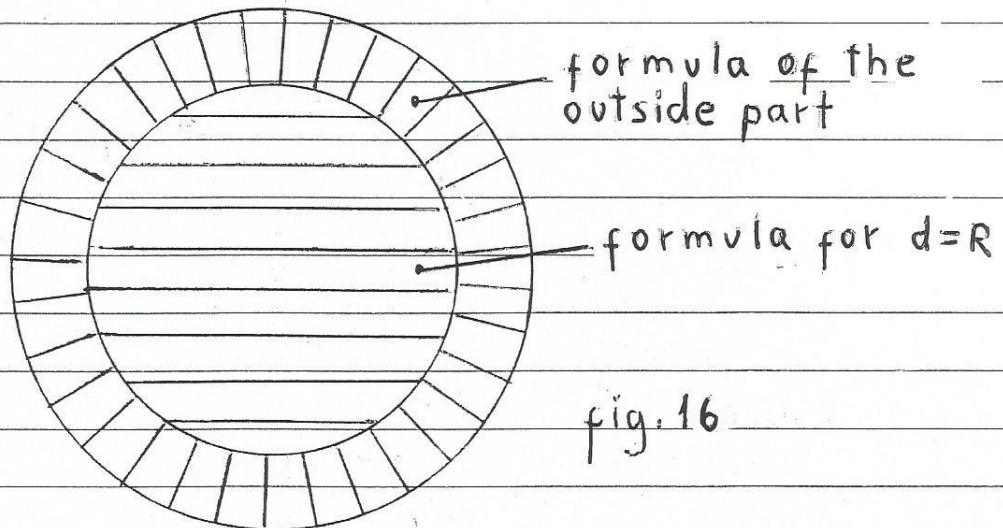
The value is very similar to the integrated formula, so it is correct.

Below is the excel spreadsheet.

d	E_0	d	E_0	d	E_0	d	E_0
64.95	38835.19943	61.15	39934.01795	57.35	41513.59623	53.55	44072.05098
64.85	38859.89710	61.05	39968.20760	57.25	41564.88967	53.45	44163.34696
64.75	38884.78919	60.95	40002.72168	57.15	41616.81832	53.35	44256.46397
64.65	38909.87808	60.85	40037.56514	57.05	41669.39575	53.25	44351.47435
64.55	38935.16618	60.75	40072.74306	56.95	41722.63595	53.15	44448.45527
64.45	38960.65594	60.65	40108.26064	56.85	41776.55338	53.05	44547.48918
64.35	38986.34987	60.55	40144.12318	56.75	41831.16299	52.95	44648.66440
64.25	39012.25050	60.45	40180.33610	56.65	41886.48025	52.85	44752.07569
64.15	39038.36043	60.35	40216.90497	56.55	41942.52114	52.75	44857.82497
64.05	39064.68228	60.25	40253.83547	56.45	41999.30224	52.65	44966.02214
63.95	39091.21874	60.15	40291.13342	56.35	42056.84067	52.55	45076.78595
63.85	39117.97253	60.05	40328.80479	56.25	42115.15422	52.45	45190.24515
63.75	39144.94645	59.95	40366.85566	56.15	42174.26128	52.35	45306.53970
63.65	39172.14331	59.85	40405.29230	56.05	42234.18096	52.25	45425.82225
63.55	39199.56600	59.75	40444.12111	55.95	42294.93307	52.15	45548.25987
63.45	39227.21746	59.65	40483.34863	55.85	42356.53820	52.05	45674.03614
63.35	39255.10067	59.55	40522.98161	55.75	42419.01770	51.95	45803.35358
63.25	39283.21869	59.45	40563.02694	55.65	42482.39379	51.85	45936.43669
63.15	39311.57462	59.35	40603.49167	55.55	42546.68959	51.75	46073.53555
63.05	39340.17163	59.25	40644.38306	55.45	42611.92914	51.65	46214.93030
62.95	39369.01294	59.15	40685.70855	55.35	42678.13748	51.55	46360.93677
62.85	39398.10184	59.05	40727.47576	55.25	42745.34070	51.45	46511.91349
62.75	39427.44168	58.95	40769.69252	55.15	42813.56602	51.35	46668.27072
62.65	39457.03587	58.85	40812.36688	55.05	42882.84185	51.25	46830.48217
62.55	39486.88791	58.75	40855.50708	54.95	42953.19784	51.15	46999.10051
62.45	39517.00134	58.65	40899.12160	54.85	43024.66500	51.05	47174.77838
62.35	39547.37979	58.55	40943.21916	54.75	43097.27576	50.95	47358.29746

62.25	39578.02696	58.45	40987.80870	54.65	43171.06406	50.85	47550.61003
62.15	39608.94662	58.35	41032.89943	54.55	43246.06549	50.75	47752.90036
62.05	39640.14262	58.25	41078.50081	54.45	43322.31736	50.65	47966.67937
61.95	39671.61889	58.15	41124.62257	54.35	43399.85884	50.55	48193.93896
61.85	39703.37943	58.05	41171.27472	54.25	43478.73111	50.45	48437.42131
61.75	39735.42834	57.95	41218.46756	54.15	43558.97746	50.35	48701.13637
61.65	39767.76981	57.85	41266.21171	54.05	43640.64353	50.25	48991.50223
61.55	39800.40809	57.75	41314.51809	53.95	43723.77739	50.15	49320.48720
61.45	39833.34754	57.65	41363.39795	53.85	43808.42981	50.05	49719.40959
61.35	39866.59262	57.55	41412.86289	53.75	43894.65445	Total E= 6325726.79000	
61.25	39900.14788	57.45	41462.92487	53.65	43982.50811		

Now let's add up the two formulas for $d = R$ and the outside part (figure 16).



$$E = \frac{\pi S}{2V_c} \left[0.5 \ln(R+d) (-R^4 + 2R^2 d^2 - d^4) + 0.5 \ln(d-R) (R^4 - 2R^2 d^2 + d^4) + R^3 d + R d^3 \right]$$

for $d=R$

$$E = \frac{\pi S}{2V_c} \left[0.5 \ln(R+R) (-R^4 + 2R^2 R^2 - R^4) + 0.5 \ln(R-R) (R^4 - 2R^2 R^2 + R^4) + R^3 R + R R^3 \right]$$

$$E = \frac{\pi S}{2V_c} [0.5 \ln 2R(0) + 0.5 \ln(0)(0) + R^4 + R^4]$$

$$E = \frac{\pi S}{2V_c} 2R^4$$

Adding the two formulas.

$$E = \frac{\pi S}{2V_c} [0.5 \ln(R+d)(-R^4 + 2R^2d^2 - d^4) + 0.5 \ln(d-R)(R^4 - 2R^2d^2 + d^4) + R^3d + Rd^3]$$

sphere formula for $d > R$

The formula for $d > R$ is practically the same formula for the internal sphere ($d \leq R$) differing only in item $\ln(R-d)$ and $\ln(d-R)$. If to make the formula of the internal sphere $\ln(|R-d|)$, it becomes a unique formula.

$$E = \frac{\pi S}{2V_c} [0.5 \ln(R+d)(-R^4 + 2R^2d^2 - d^4) + 0.5 \ln(|R-d|)(R^4 - 2R^2d^2 + d^4) + R^3d + Rd^3]$$

sphere formula

From page 5 to this page (65), the objective was to deduce the disk formula and the sphere formula.

CALCULATION OF THE SPEED FOR THE GALAXY ROTATION CURVE

The nucleus and the disk are not uniform, the energy density is variable, and so they were divided into sections that are approximately uniform, providing a good accuracy for the calculations.

The data were obtained from the graphic on page 3, the coordinates were taken very accurate and cautiously.

(1) Table of visible mass of the nucleus

dist. (pc)	dist.(m)	V_v (Km/s)	M_v (Kg)
250	7.71420E+18	237	6.4962353E+39
500	1.54284E+19	244	1.3771292E+40
750	2.31426E+19	240	1.9985213E+40
1000	3.08568E+19	230	2.4472634E+40
1250	3.85710E+19	221	2.8243571E+40
1500	4.62852E+19	212	3.1188036E+40
1750	5.39994E+19	207	3.4689959E+40
2000	6.17136E+19	200	3.7009655E+40

(2) Table of visible mass of the disk

dist. (pc)	dist.(m)	V_v (Km/s)	M_v (Kg)
2000	6.17136E+19	200	3.7009655E+40
3100	9.565608E+19	182	4.7503927E+40
4050	1.2497004E+20	172.5	5.5751720E+40
5000	1.54284E+20	167	6.4510142E+40
6250	1.92855E+20	162	7.5881358E+40
7500	2.31426E+20	159	8.7716352E+40
8750	2.69997E+20	155	9.7251543E+40
10000	3.08568E+20	151	1.0548214E+41
11250	3.47139E+20	148	1.1399899E+41
12500	3.85710E+20	145	1.2158250E+41
13750	4.24281E+20	141	1.2646372E+41
15330	4.73037E+20	136	1.3117379E+41

dist. (pc) - distance to the galaxy center in parsec

dist. (m) - distance to the galaxy center in meters

Vv - speed of visible matter in km / s

Mv - mass of the visible matter in Kg

The distance to the galaxy center (pc) and the speed of the visible matter in km / s were obtained from the graph on page 3. The mass of visible matter was calculated using the formula $M = V^2 \cdot d / G$, let's see an example for the distance of 250 pc in the nucleus.

$$d = 7.7142 \times 10^{18} \text{ m}$$

$$V = 237 \text{ km/s} = 237000 \text{ m/s}$$

$$G = 6.67 \times 10^{-11} \text{ N.m}^2/\text{Kg}^2$$

$$M = \frac{V^2 \cdot d}{G} = \frac{237000^2 \times 7.7142 \times 10^{18}}{6.67 \times 10^{-11}} = 6.4962353 \times 10^{39} \text{ Kg}$$

(3) Table of energy density of the nucleus

Section(pc)	Mv(Kg)	volume(m ³)	dens.(Kg/m ³)	S(w/m ³)
0 - 250	6.4962353E+39	1.9229203E+57	3.3783174E-18	1.0528015E-21
250 - 500	7.2750567E+39	1.3460442E+58	5.4047679E-19	1.6843142E-22
500 - 750	6.213921E+39	3.6535486E+58	1.7007905E-19	5.3002566E-23
750 - 1000	4.487421E+39	7.1148052E+58	6.3071592E-20	1.9655308E-23
1000 - 1250	3.770937E+39	1.1729814E+59	3.2148310E-20	1.0018535E-23
1250 - 1500	2.944465E+39	1.7498575E+59	1.6826884E-20	5.2438441E-24
1500 - 1750	3.501923E+39	2.4421088E+59	1.4339749E-20	4.4687661E-24
1750 - 2000	2.319696E+39	3.2497353E+59	7.1381073E-21	2.2244832E-24

Mv - mass of visible matter in the section

volume (m³) - volume of the section

dens. (Kg / m³) - volumetric density of the section

S (w / m³) - energy density of the section

The mass of the visible matter of the section was obtained by subtracting the values in the previous table, we will see an example of the section 250-500.

$$Mv = 1.3771292 \times 10^{40} - 6.4962353 \times 10^{39} \quad Mv = 7.2750567 \times 10^{39} \text{ Kg}$$

The volume was calculated using the formula $Vol = 4\pi (R^3 - r^3) / 3$, where the radius is the distance d.

The volumetric density was calculated by dividing the mass by the volume.

The energy density is proportional to the volumetric density, we will see an example.

Galaxy power: 10^{11} suns

Nucleus power: 0.3×10^{11} suns

Sun power: 3.84451×10^{26} w

Nucleus power: $0.3 \times 10^{11} \times 3.84451 \times 10^{26} = 1.15335 \times 10^{37}$ w

Mass of the nucleus: 3.7009655×10^{40} Kg

$$D = \frac{P}{m} \quad D = \frac{1.15335 \times 10^{37}}{3.7009655 \times 10^{40}} \quad D = 3.1163489 \times 10^{-4} \text{ w/Kg}$$

For section 0 – 250

$$S = D \cdot \text{dens} \quad S = 3.1163489 \times 10^{-4} \times 3.3783174 \times 10^{-18} \quad S = 1.0528015 \times 10^{-21} \text{ w/m}^3$$

(4) Table of the energy density of disk

section (pc)	Mv(Kg)	area(m ²)	dens.(Kg/m ²)	s(W/m ²)
2000 - 3100	1.0494272E+40	1.6780871E+40	0.625371114	1.78727679E-04
3100 - 4050	8.2477930E+39	2.0318015E+40	0.405934979	1.16014019E-04
4050 - 5000	8.7584220E+39	2.5717208E+40	0.340566596	9.73320893E-05
5000 - 6250	1.1371216E+40	4.2064349E+40	0.270329061	7.72585821E-05
6250 - 7500	1.1834994E+40	5.1411982E+40	0.230199139	6.57896676E-05
7500 - 8750	9.5351910E+39	6.0759616E+40	0.156933035	4.48506118E-05
8750 - 10000	8.2305970E+39	7.0107249E+40	0.117400085	3.35523087E-05
10000 - 11250	8.5168500E+39	7.9454882E+40	0.107191021	3.06346134E-05
11250 - 12500	7.5835100E+39	8.8802516E+40	0.085397467	2.44061337E-05
12500 - 13750	4.8812200E+39	9.8150149E+40	0.049732171	1.42131852E-05
13750 - 15330	4.7100700E+39	1.3744353E+41	0.034269128	9.79393120E-06

Mv - mass of visible matter in the section
 area (m²) = area of the section
 dens. (Kg / m²) - surface density of the section
 S (w / m²) - energy density of the section

The mass of the visible matter of the section was obtained by subtracting the values in the table 2, we will see an example of the section 2000 - 3100.

$$Mv = 4.7503927 \times 10^{40} - 3.7009655 \times 10^{40} \quad Mv = 1.0494272 \times 10^{40} \text{ Kg}$$

The area was calculated using the formula $A = \pi (R^2 - r^2)$, where the radius is the distance d.

The surface density was calculated by dividing the mass by the area.

The energy density is proportional to the surface density, we will see an example.

Galaxy power: 10^{11} suns

Disk power: 0.7×10^{11} suns

Sun power: 3.84451×10^{26} w

Disk power: $0.7 \times 10^{11} \times 3.84451 \times 10^{26} = 2.69116 \times 10^{37}$ w

Mass of the disk: $1.3117379 \times 10^{41} - 3.7009655 \times 10^{40} = 9.4164135 \times 10^{40}$ Kg

$$D = \frac{P}{m} \quad D = \frac{2.69116 \times 10^{37}}{9.4164135 \times 10^{40}} \quad D = 2.857945862 \times 10^{-4} \text{ w/Kg}$$

For section 2000 - 3100

$$s = D \cdot \text{dens} \quad s = 2.857945862 \times 10^{-4} \times 0.625371114 \quad s = 1.787276788 \times 10^{-4} \text{ w/m}^2$$

The galaxy rotation curve was divided into 14 orbits, with distance in parsec: 500, 2000 (end of the nucleus), 3100, 5000, 7500, 10000, 12500, 15330 (end of the disk), 17500, 20000, 22500, 25000, 27500, 30000.

The following table is for determining the energy contained within each orbit, to do this will use the formula of light emitting sphere, to calculate the energy emission of the nucleus, and the formula of the light emitting disk, to calculate the energy emission of disk, adding those two is the total of energy.

(5) Table of the energy within the orbits

radius of orbit (pc)	E_{nucleus} (joule)	E_{disk} (joule)	E_{total} (joule)
500	2.2579504E+47	4.6175078E+45	2.3041254E+47
2000	2.1510818E+48	3.2843158E+47	2.4795133E+48
3100	3.5506865E+48	1.3683228E+48	4.9190093E+48
5000	5.8577508E+48	4.6559081E+48	1.0513658E+49
7500	8.8518210E+48	1.1231850E+49	2.0083671E+49
10000	1.1832441E+49	1.9276375E+49	3.1108816E+49
12500	1.4807829E+49	2.8018880E+49	4.2826709E+49
15330	1.8173002E+49	3.7641904E+49	5.5814906E+49
17500	2.0751962E+49	4.4444355E+49	6.5196317E+49
20000	2.3722263E+49	5.1968736E+49	7.5690999E+49
22500	2.6692185E+49	5.9325406E+49	8.6017591E+49
25000	2.9661566E+49	6.6579843E+49	9.6241409E+49
27500	3.2630638E+49	7.3766201E+49	1.0639683E+50
30000	3.5599477E+49	8.0904303E+49	1.1650378E+50

To do the calculations mentioned above, it was used two excel spreadsheet, one for the nucleus and another for the disk. The disk and the nucleus were divided into sections with different energy density then the energy was calculated in each section, we will see an example in the orbit of radius 30000pc (9.25704×10^{20} m), the energy radiated by the nucleus: In the section 250-500, I have to calculate the energy to 500pc (1.54284×10^{19} m) and decrease the energy of 250pc (7.7142×10^{18} m), and like the same way in the other sections. The first 8 calculations of energy is for the positive part and the other next 7 is for the negative part, then made to sum up of the positive part and the negative part. Subtracting the negative part from the positive part I get the energy radiated by the nucleus within that orbit.

The excel spreadsheet to the nucleus is in Annex I, and to the disk is in Annex II.

THE GRAVITATION CONSTANT OF ELECTROMAGNETIC WAVES

To calculate the speed in the orbits it is necessary to have gravitational

constant of electromagnetic waves, which have a tendency to be different and higher than the universal gravitational constant $G = 6.67 \times 10^{-11} \text{ N.m}^2 / \text{Kg}^2$.

Matter is made of atoms, but not electromagnetic waves, it is an oscillating electric fields and magnetic fields (its particles were called photons), it is expected that its gravitational constant is much higher.

Let's calculate the gravitational constant of the electromagnetic waves (light), using the speed of the last orbit of the graph of radius is equal to 30000 pc, what will be used the formula deduced below:

The gravitational force of attraction will be the sum of the force caused by the visible matter more the force caused by the electromagnetic waves.

$$F = \frac{G.M.m}{d^2} + \frac{G_e.Me.m}{d^2} \quad F = \frac{m.V^2}{d} \quad \frac{m.V^2}{d} = \frac{m.(G.M + G_e.Me)}{d^2}$$

$V^2 = \frac{G.M + G_e.Me}{d}$	$G_e = \frac{V^2.d - G.M}{Me}$
--------------------------------	--------------------------------

G_e – gravitation constant of electromagnetic waves

V - speed in the orbit

d - radius of the orbit

G - universal gravitational constant

M - mass of visible matter

Me - mass of electromsgnetic waves

For orbit of 30000 pc

V – 211Km/s - 211000m/s

d - 30000 pc - 9.25704×10^{20} m

G - $6.67 \times 10^{-11} \text{ N.m}^2/\text{Kg}^2$

M - $1.4326289 \times 10^{41} \text{ Kg}$

Me – $1.2962793 \times 10^{33} \text{ Kg}$

The Me was calculated by the formula $E = m.Vc^2$, where E is the energy in the orbit of 30000 pc.

$$1.1650378 \times 10^{50} = m \times 299792458^2 \quad m = 1.2962793 \times 10^{33} \text{ Kg}$$

$$G_e = \frac{211000^2 \times 9.25704 \times 10^{20} - 6.67 \times 10^{-11} \times 1.4326289^{41}}{1.2962793 \times 10^{33}}$$

$$G_e = 0.024421922 \text{ N.m}^2/\text{Kg}^2$$

By doing the calculation of G_e , we will calculate the speeds in 13 other orbits, to see if the value is equal to the observed speeds.

Below is a table (6) that shows, the speeds in the orbits.

A	B	C	D	E	F	G	H
500	1.54284E+19	244	1.3771292E+40	2.5636853E+30	252.179	246	6.179
2000	6.17136E+19	200	3.7009655E+40	2.7588307E+31	225.649	219	6.649
3100	9.565608E+19	182	4.7503927E+40	5.4731359E+31	217.019	215	2.019
5000	1.54284E+20	167	6.4510142E+40	1.1698023E+32	215.421	219	-3.579
7500	2.31426E+20	159	8.7716352E+40	2.2346097E+32	221.048	223	-1.952
10000	3.08568E+20	151	1.0548214E+41	3.4613225E+32	224.045	225	-0.955
12500	3.85710E+20	145	1.2158250E+41	4.7651140E+32	226.266	225	1.266
15330	4.73037E+20	136	1.3117379E+41	6.2102458E+32	224.852	222	2.852
17500	5.39994E+20	130	1.3682006E+41	7.2540685E+32	222.952	221	1.952
20000	6.17133E+20	124	1.4226442E+41	8.4217594E+32	220.689	218	2.689
22500	6.94278E+20	117	1.4248832E+41	9.5707477E+32	217.612	216	1.612
25000	7.71420E+20	111	1.4249873E+41	1.0708300E+33	214.992	214	0.992
27500	8.48562E+20	106	1.4294516E+41	1.1838244E+33	212.854	212.5	0.354
30000	9.25704E+20	101.6	1.4326289E+41	1.2962793E+33	211.000	211	0.000

A – d(pc), radius of the orbit in parsec

B – d(m), radius of the orbit in meter

C – Vv(Km/s), speed of visible matter in Km/s

D – Mv(Kg), mass of visible matter in Kg

E – Mc(Kg), mass of electromagnetic waves in Kg

F – V(Km/s), calculated speed in Km/s

G – Vobs(Km/s), observed speed in Km/s

H - the speed difference between observed speed and calculated speed.

The M_e was calculated using the formula $E = m.Vc^2$, where E is the total energy contained in the table (5).

The speed was calculated using the formula: $V^2 = \frac{G.M + G_e.M_e}{d}$

Let us now analyze the result:

The difference between the calculated speed and the observed speed was an average of 1.838 m /s in eleven points on the disk, this is a small difference and the result was excellent. In the two orbits of the nucleus (500 and 2000), it has a greater difference in average 6.414 km / s, and this is acceptable because the curve of the visible matter was assembled by observation and counting, and not by calculation, observing the disk is easier than observing the nucleus.

Below is the graph of the observed curve and the calculated curve.

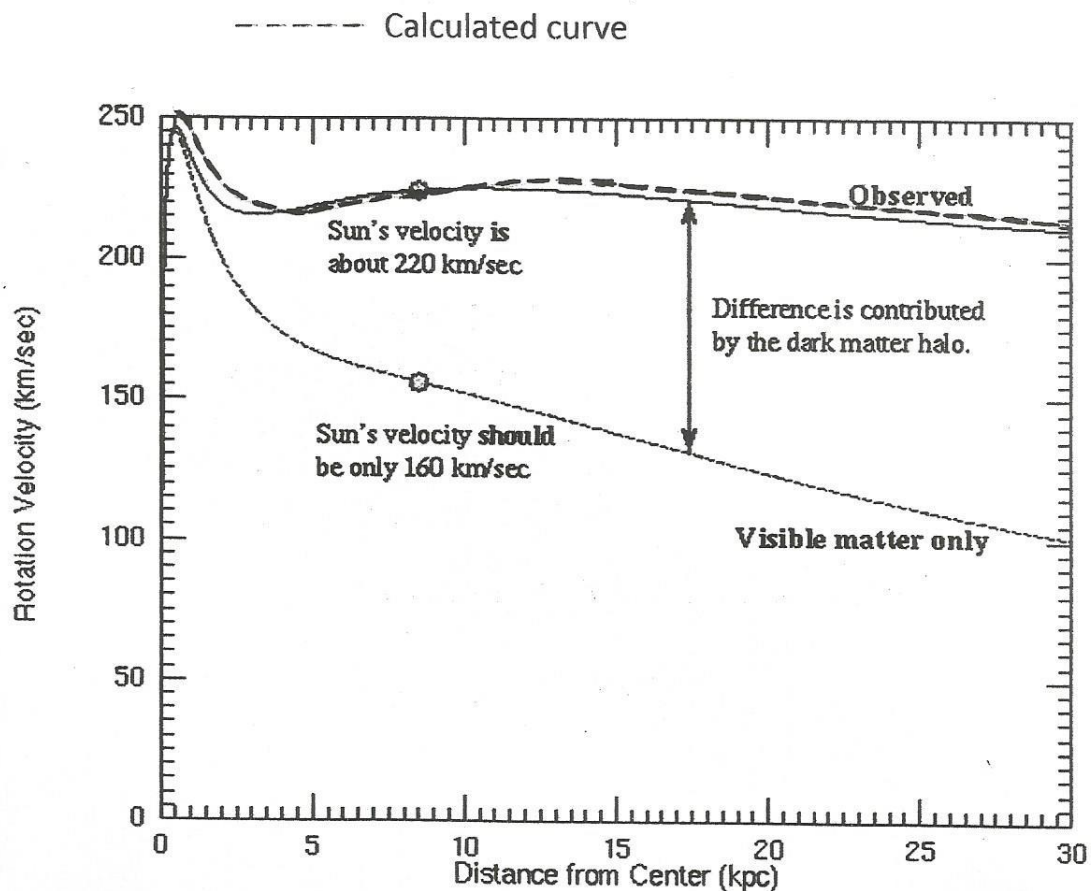


fig. 17

The influence of the electromagnetic waves (light) on the orbit of the solar system is insignificant, but when the orbits become huge, as the rotation of the Milky Way (also in other galaxies) and the orbits of galaxies in galaxy clusters, electromagnetic waves become remarkable, and the larger the orbit, the greater its influence.

The orbit of Pluto:

Mean distance from Pluto to the sun = 5913520000Km or 5.91352×10^{12} m

$$E = P.t \quad t = \frac{d}{V_c} \quad t = \frac{5.91352 \times 10^{12}}{299792458} \quad t = 19725.37948s$$

Sun power: 3.84451×10^{26} w

$$E = P.t \quad E = 3.84451 \times 10^{26} \times 19725.37948 \quad E = 7.5834418 \times 10^{30} \text{ J}$$

$$E = m.V_c^2 \quad m = \frac{E}{V_c^2} \quad m = \frac{7.5834418 \times 10^{30}}{299792458^2} \quad m = 8.4377169 \times 10^{13} \text{ Kg}$$

We call it gravitational mass, the mass times its gravitational constant, its unit is $N.m^2 / kg$.

Then the gravitational mass of the electromagnetic waves in Pluto's orbit is:

$$mg = 8.4377169 \times 10^{13} \times 0.024421922 \quad mg = 2.0606526 \times 10^{12} \text{ N.m}^2/\text{Kg}$$

The gravitational mass of the sun is::

$$mg = 1.98892 \times 10^{30} \times 6.67 \times 10^{-11} \quad mg = 1.3266096 \times 10^{20} \text{ N.m}^2/\text{Kg}$$

$$X = \frac{1.3266096 \times 10^{20}}{2.0606526 \times 10^{12}} \quad X = 6.44 \times 10^7$$

The gravitational mass of the sun is 6.44×10^7 greater than the gravitational mass of the electromagnetic waves. We will now see the influence of the electromagnetic waves at the speed of Pluto's orbit.

Only the mass of the sun:

$$V^2 = \frac{G.M}{d} \quad V^2 = \frac{6.67 \times 10^{-11} \times 1.98892 \times 10^{30}}{5.91352 \times 10^{12}} \quad V = 4736.401821 \text{ m/s}$$

Now with the mass of the electromagnetic waves:

$$V^2 = \frac{G.M + G_e.M_e}{d}$$

$$V^2 = \frac{6.67 \times 10^{-11} \times 1.98892 \times 10^{30} + 0.024421922 \times 8.4377169 \times 10^{13}}{5.91352 \times 10^{12}}$$

$$V = 4736.401858 \text{ m/s}$$

In calculating the speed, it results in an insignificant difference 0.000037 m/s .

We will now see the influence of the spin on the edge of the disk of the Milky Way. The radius of the galactic disk is 15330 pc or $4.73037 \times 10^{20} \text{ m}$.

On table 6, which was made to calculate the galaxy's rotation took the mass of the visible matter which is $1.3117379 \times 10^{41} \text{ kg}$ and the mass of the electromagnetic waves that is $6.2102458 \times 10^{32} \text{ Kg}$, let's calculate its gravitational masses.

$$\text{Gravitational mass of the visible matter: } mg = 1.3117379 \times 10^{41} \times 6.67 \times 10^{-11}$$

$$mg = 8.7492917 \times 10^{30} \text{ N.m}^2/\text{Kg}$$

Gravitational mass of the electromagnetic waves:

$$mg = 6.2102458 \times 10^{32} \times 0.024421922 \quad mg = 1.5166613 \times 10^{31} \text{ N.m}^2/\text{Kg}$$

$$X = \frac{1.5166613 \times 10^{31}}{8.7492917 \times 10^{30}} \quad X = 1.733$$

The gravitational mass of the electromagnetic waves is 1.733 times that of the visible matter.

We will now see the influence of the electromagnetic waves in the orbit of the galaxies in the cluster virgin, where the radius of the orbits are very large.

$$\text{Radius} = 1.5 \times 10^6 \text{ pc} - 4.6285 \times 10^{22} \text{ m}$$

$$\text{Mass} = 1 \times 10^{14} \text{ suns} - 1.98892 \times 10^{44} \text{ Kg}$$

$$\text{Power} = 1 \times 10^{14} \text{ suns} - 3.84451 \times 10^{40} \text{ w}$$

In the extremity 50% of the electromagnetic energy is internal to the orbit and the center 100% of the energy is internal, then we multiply by 0.75 as an approximate average.

$$P = 3.84451 \times 10^{40} \times 0.75 \quad P = 2.88338 \times 10^{40} \text{ w}$$

$$E = P \cdot t \quad t = \frac{R}{Vc} \quad t = \frac{4.6285 \times 10^{22}}{299792458} \quad t = 1.5439 \times 10^{14} \text{ s}$$

$$E = 2.88338 \times 10^{40} \times 1.5439 \times 10^{14} \quad E = 4.45165 \times 10^{54} \text{ J}$$

$$E = m \cdot Vc^2 \quad m = \frac{E}{Vc^2} \quad m = \frac{4.45165 \times 10^{54}}{299792458^2} \quad m = 4.95313 \times 10^{37} \text{ Kg}$$

Let's calculate the gravitational masses:

Of the visible matter:

$$mg = 1.98892 \times 10^{44} \times 6.67 \times 10^{-11} \quad mg = 1.32661 \times 10^{34} \text{ N.m}^2/\text{Kg}$$

Of the electromagnetic waves:

$$mg = 4.95313 \times 10^{37} \times 0.024421922 \quad mg = 1.20965 \times 10^{36} \text{ N.m}^2/\text{Kg}$$

$$X = \frac{1.20965 \times 10^{36}}{1.32661 \times 10^{34}} \quad X = 91.18$$

The gravitational mass of the electromagnetic waves is 91.18 times that of the visible matter, which fits with the early work of Zwicky, where he said that dark matter was 100 times more than the visible matter, which was hard to believe.

SIMULATION

Now let's do 3 simulations, to calculate an index on the last orbit, and then calculating the other speeds, equal to what was done with the electromagnetic waves; thus analyzing the curve behavior.

SIMULATION 1

Let's imagine that the dark matter is a substance of homogeneous density distributed throughout the galaxy, with a normal gravitational constant $6.67 \times 10^{-11} \text{ N.m}^2 / \text{Kg}^2$. Let's calculate its density using the 30Kpc orbit with the speed of 211km / s, and then we calculate the speed in other points and build up the curve.

Mt – total mass

Me – mass of the dark matter

Mv – mass of the visible matter

$$Mt = \frac{V^2 \cdot d}{G} \quad V = 211 \text{ Km/s} = 2.11 \times 10^5 \text{ m/s} \quad d = 9.25704 \times 10^{20} \text{ m} \\ G = 6.67 \times 10^{-11} \text{ N.m}^2/\text{Kg}^2$$

$$Mt = \frac{(2.11 \times 10^5)^2 \times 9.25704 \times 10^{20}}{6.67 \times 10^{-11}} \quad Mt = 6.1789007 \times 10^{41} \text{ Kg}$$

$$Me = Mt - Mv \quad Me = 6.1789007 \times 10^{41} - 1.4326289 \times 10^{41}$$

$$M_e = 4.7462718 \times 10^{41} \text{ Kg}$$

$$\text{Sphere volume} = \frac{4\pi R^3}{3} \quad \text{Vol} = \frac{4}{3} \times \pi \times (9.25704 \times 10^{20})^3$$

$$\text{Vol} = 3.3228063 \times 10^{63} \text{ m}^3$$

$$\text{den} = \frac{m}{\text{Vol}} \quad \text{den} = \frac{4.7462718 \times 10^{41}}{3.3228063 \times 10^{63}} \quad \text{den} = 1.4283925 \times 10^{-22} \text{ Kg/m}^3$$

Let's calculate the speed with the excel spreadsheet attached.

A	B	C	D	E	F	G	H
500	1.542840E+19	1.3771292E+40	2.1973480E+36	1.3773489E+40	244.019	246	-1.981
2000	6.171360E+19	3.7009655E+40	1.4063027E+38	3.7150285E+40	200.380	219	-18.620
3100	9.565608E+19	4.7503927E+40	5.2368955E+38	4.8027617E+40	183.000	215	-32.000
5000	1.542840E+20	6.4510142E+40	2.1973480E+39	6.6707490E+40	169.820	219	-49.180
7500	2.314260E+20	8.7716352E+40	7.4160495E+39	9.5132401E+40	165.585	223	-57.415
10000	3.085680E+20	1.0548214E+41	1.7578784E+40	1.2306092E+41	163.098	225	-61.902
12500	3.857100E+20	1.2158250E+41	3.4333562E+40	1.5591606E+41	164.202	225	-60.798
15330	4.730370E+20	1.3117379E+41	6.3331752E+40	1.9450554E+41	165.608	222	-56.392
17500	5.399940E+20	1.3682006E+41	9.4211295E+40	2.3103135E+41	168.929	221	-52.071
20000	6.171330E+20	1.4226442E+41	1.4062822E+41	2.8289264E+41	174.858	218	-43.142
22500	6.942780E+20	1.4248832E+41	2.0023334E+41	3.4272166E+41	181.454	216	-34.546
25000	7.714200E+20	1.4249873E+41	2.7466850E+41	4.1716723E+41	189.921	214	-24.079
27500	8.485620E+20	1.4294516E+41	3.6558377E+41	5.0852893E+41	199.930	212.5	-12.570
30000	9.257040E+20	1.4326289E+41	4.7462717E+41	6.1789006E+41	211.000	211	0.000

A – d(pc), radius of the orbit in parsec

B – d(m), radius of the orbit in meter

C – Mv(Kg), mass of visible matter in Kg

D – Me(Kg), mass of dark matter in Kg

E – Mt(Kg), mass of total matter in Kg

F – V(Km/s), calculated speed in Km/s

G – Vobs(Km/s), observed speed in Km/s

H - the speed difference between observed speed and calculated speed.

The calculation of M_e was made by the formula:

$$M_e = \text{Vol} \times \text{den} \quad M_e = \frac{4}{3} \times \pi \times d^3 \times 1.4283925 \times 10^{-22}$$

The calculation of speed was made by the formula:

$$V^2 = \frac{G \times M_t}{d}$$

Below is the graph of the calculated curve, and we see that it deviates from its path.

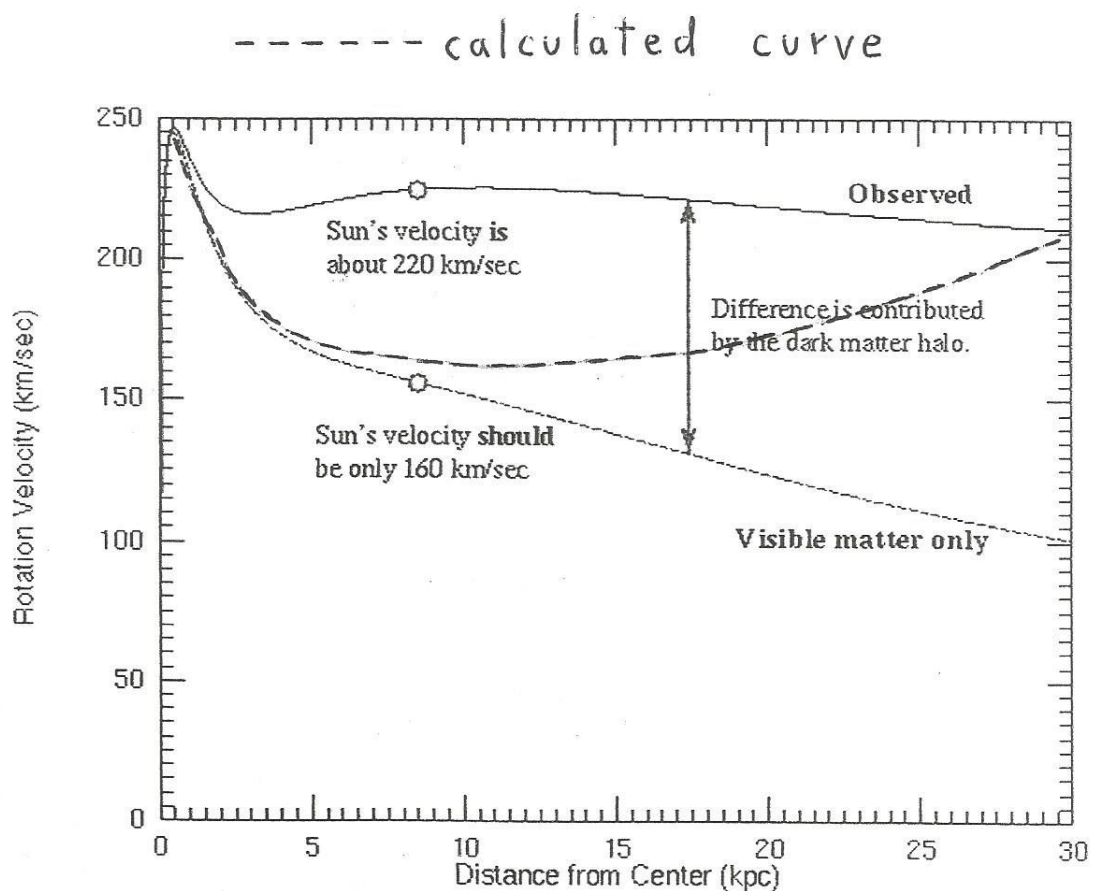


fig 18

SIMULATION 2

This simulation is the same as the simulation 1, just changing the constant of gravitation which is a geometric mean between the universal gravitational constant $6.67 \times 10^{-11} \text{ N.m}^2 / \text{Kg}^2$ and the gravitational constant of electromagnetic waves $0.024421922 \text{ N.m}^2 / \text{Kg}^2$, resulting in $1.2763 \times 10^{-6} \text{ N.m}^2 / \text{Kg}^2$.

$$V^2 = \frac{G.Mv + Ge.Me}{d} \quad Me = \frac{d.V^2 - G.Mv}{Ge}$$

$$d = 9.25704 \times 10^{20} \text{ m} \quad V = 2.11 \times 10^5 \text{ m/s} \quad G = 6.67 \times 10^{-11} \text{ N.m}^2/\text{Kg}^2$$

$$Mv = 1.4326289 \times 10^{41} \text{ Kg} \quad Ge = 1.2763 \times 10^{-6} \text{ N.m}^2/\text{Kg}^2$$

$$Me = \frac{9.25704 \times 10^{20} \times (2.11 \times 10^5)^2 - 6.67 \times 10^{-11} \times 1.4326289 \times 10^{41}}{1.2763 \times 10^{-6}}$$

$$Me = 2.4804225 \times 10^{37} \text{ Kg}$$

$$\text{Vol} = \frac{4\pi R^3}{3} \quad \text{Vol} = \frac{4}{3} \times \pi \times (9.25704 \times 10^{20})^3 \quad \text{Vol} = 3.3228063 \times 10^{63} \text{ m}^3$$

$$\text{den} = \frac{m}{\text{Vol}} \quad \text{den} = \frac{2.4804225 \times 10^{37}}{3.3228063 \times 10^{63}} \quad \text{den} = 7.4648422 \times 10^{-27} \text{ Kg/m}^3$$

Let's calculate the speed with the excel spreadsheet attached.

A	B	C	D	E	F	G
500	1.542840E+19	1.3771292E+40	1.1483437E+32	244.019	246	-1.981
2000	6.171360E+19	3.7009655E+40	7.3494000E+33	200.380	219	-18.620
3100	9.565608E+19	4.7503927E+40	2.7368247E+34	183.000	215	-32.000
5000	1.542840E+20	6.4510142E+40	1.1483437E+35	169.820	219	-49.180
7500	2.314260E+20	8.7716352E+40	3.8756602E+35	165.585	223	-57.415
10000	3.085680E+20	1.0548214E+41	9.1867500E+35	163.098	225	-61.902
12500	3.857100E+20	1.2158250E+41	1.7942871E+36	164.202	225	-60.798
15330	4.730370E+20	1.3117379E+41	3.3097453E+36	165.608	222	-56.392
17500	5.399940E+20	1.3682006E+41	4.9235238E+36	168.929	221	-52.071
20000	6.171330E+20	1.4226442E+41	7.3492928E+36	174.858	218	-43.142
22500	6.942780E+20	1.4248832E+41	1.0464282E+37	181.454	216	-34.546
25000	7.714200E+20	1.4249873E+41	1.4354297E+37	189.921	214	-24.079
27500	8.485620E+20	1.4294516E+41	1.9105569E+37	199.930	212.5	-12.570
30000	9.257040E+20	1.4326289E+41	2.4804225E+37	211.000	211	0.000

A – d(pc), radius of the orbit in parsec

B – d(m), radius of the orbit in meter

C – Mv(Kg), mass of visible matter in Kg

D – Me(Kg), mass of dark matter in Kg

E – V(Km/s), calculated speed in Km/s

F – Vobs(Km/s), observed speed in Km/s

G - the speed difference between observed speed and calculated speed.

The calculation of Me was made by this formula:

$$Me = Vol \times den \quad Me = \frac{4}{3} \times \pi \times d^3 \times 7.4648422 \times 10^{-27}$$

The calculation of speed was made by this formula:

$$V^2 = \frac{G \times Mv + G \times Me}{d}$$

Analyzing the values of speed, we see that it was identical to the simulation 1, and the curve will also be identical, which seems to not depend on the gravitational constant of the dark matter, then we will make a generic formula to check the situation.

₃₀ – is the values that corresponds to the orbit of 30Kpc.

$$V^2 = \frac{G \cdot Mv + G \cdot Me}{d} \quad Me = Vol \cdot den \quad den = \frac{Me_{30}}{Vol_{30}}$$

$$Me_{30} = \frac{d_{30} \cdot V_{30}^2 - G \cdot Mv_{30}}{Ge} \quad den = \frac{d_{30} \cdot V_{30}^2 - G \cdot Mv_{30}}{Ge \cdot Vol_{30}}$$

$$Me = \frac{Vol \cdot (d_{30} \cdot V_{30}^2 - G \cdot Mv_{30})}{Ge \cdot Vol_{30}} \quad V^2 = \frac{G \cdot Mv + \cancel{Ge} \cdot Vol \cdot (d_{30} \cdot V_{30}^2 - G \cdot Mv_{30})}{\cancel{Ge} \cdot Vol_{30} \cdot d}$$

$$V^2 = \frac{G \cdot Mv}{d} + \frac{Vol \cdot (d_{30} \cdot V_{30}^2 - G \cdot Mv_{30})}{d \cdot Vol_{30}}$$

The Ge was cut, so the speed does not depend on the gravitational constant of dark matter. Thus it is impossible that the dark matter is distributed by the homogeneous manner in the galaxy.

SIMULATION 3

Let's imagine that dark matter is an irradiated substance just as electromagnetic waves, but its radiation is concentrated in the center of the galaxy, and that the gravitational constant is equal to the electromagnetic waves $0.024421922 \text{ N.m}^2 / \text{Kg}^2$.

Let's calculate the radiation flow to the 30Kpc orbit fits in its speed, then we calculate the speed in other points and build up the curve.

$$V^2 = \frac{G.Mv + Ge.Me}{d} \quad Me = \frac{d.V^2 - G.Mv}{Ge}$$

$$d = 9.25704 \times 10^{20} \text{ m} \quad V = 2.11 \times 10^5 \text{ m/s} \quad G = 6.67 \times 10^{-11} \text{ N.m}^2/\text{Kg}^2$$

$$Mv = 1.4326289 \times 10^{41} \text{ Kg} \quad Ge = 0.024421922 \text{ N.m}^2/\text{Kg}^2$$

$$Me = \frac{9.25704 \times 10^{20} \times (2.11 \times 10^5)^2 - 6.67 \times 10^{-11} \times 1.4326289 \times 10^{41}}{0.024421922}$$

$$Me = 1.2962793 \times 10^{33} \text{ Kg}$$

The calculation of the matter flow will be similar to that of the energy.

$$E = P.t \quad t = \frac{d}{Vc} \quad E = \frac{P.d}{Vc}$$

In place of E (energy) will be Me (mass), and in place of P (power) is F (flow).

$$Me = F.t \quad t = \frac{d}{Vc} \quad Me = \frac{F.d}{Vc}$$

$$Me = 1.2962793 \times 10^{33} \text{ Kg} \quad d = 9.25704 \times 10^{20} \text{ m} \quad Vc = 299792458 \text{ m/s}$$

$$1.2962793 \times 10^{33} = \frac{F \times 9.25704 \times 10^{20}}{299792458} \quad F = 4.1980455 \times 10^{20} \text{ Kg/s}$$

Let's calculate the speed with the excel spreadsheet attached.

A	B	C	D	E	F	G
500	1.542840E+19	1.3771292E+40	2.1604655E+31	306.161	246	60.161
2000	6.171360E+19	3.7009655E+40	8.6418619E+31	272.394	219	53.394
3100	9.565608E+19	4.7503927E+40	1.3394886E+32	259.466	215	44.466
5000	1.542840E+20	6.4510142E+40	2.1604655E+32	249.174	219	30.174
7500	2.314260E+20	8.7716352E+40	3.2406982E+32	243.884	223	20.884
10000	3.085680E+20	1.0548214E+41	4.3209309E+32	238.746	225	13.746
12500	3.857100E+20	1.2158250E+41	5.4011637E+32	234.997	225	9.997
15330	4.730370E+20	1.3117379E+41	6.6240187E+32	229.553	222	7.553
17500	5.399940E+20	1.3682006E+41	7.5616291E+32	226.050	221	5.050
20000	6.171330E+20	1.4226442E+41	8.6418198E+32	222.653	218	4.653
22500	6.942780E+20	1.4248832E+41	9.7220946E+32	218.832	216	2.832
25000	7.714200E+20	1.4249873E+41	1.0802327E+33	215.684	214	1.684
27500	8.485620E+20	1.4294516E+41	1.1882560E+33	213.154	212.5	0.654
30000	9.257040E+20	1.4326289E+41	1.2962793E+33	211.000	211	0.000

A – d(pc), radius of the orbit in parsec

B – d(m), radius of the orbit in meter

C – Mv(Kg), mass of visible matter in Kg

D – Me(Kg), mass of dark matter in Kg

E – V(Km/s), calculated speed in Km/s

F – Vobs(Km/s), observed speed in Km/s

G - the speed difference between observed speed and calculated speed.

The calculation of Me was made by the formula:

$$Me = \frac{F \cdot d}{V_c} \quad Me = \frac{4.1980455 \times 10^{20} \times d}{299792458} \quad Me = 1.4003172 \times 10^{12} \times d$$

The calculation of the speed was made by the formula:

$$V^2 = \frac{G \cdot M_v + G_e \cdot M_e}{d}$$

Analyzing the curve, we see that it has deviated from the path.

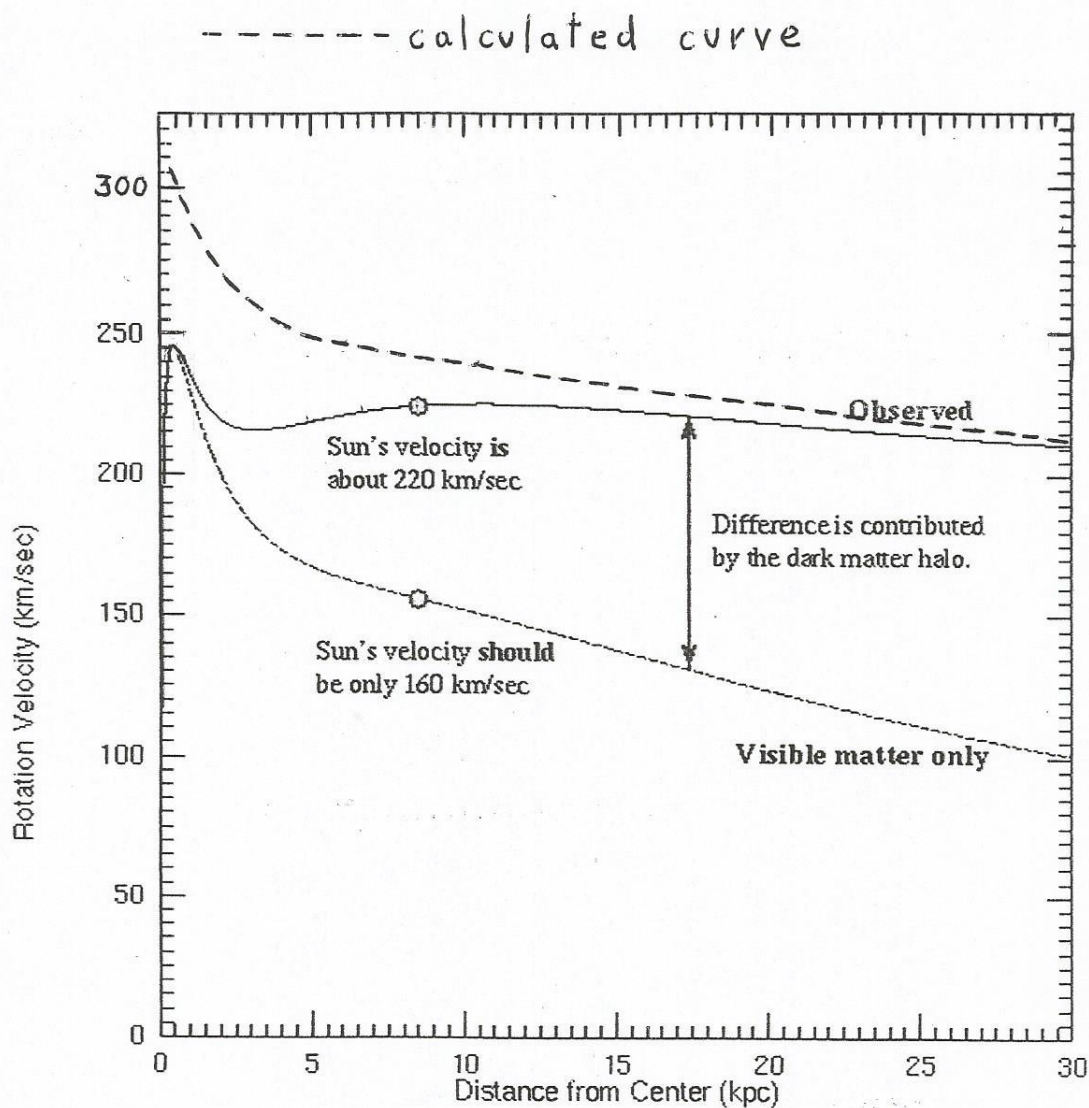


fig. 19

Let's make a generic formula to see if the speeds will depend on the gravitational constant of dark matter.

$$V^2 = \frac{G.Mv + G_e.M_e}{d} \quad M_e = \frac{F.d}{V_c} \quad V^2 = \frac{G.Mv + \frac{G_e.F.d}{V_c}}{d}$$

$$M_{e30} = \frac{d_{30}.V_{30}^2 - G.Mv_{30}}{G_e} \quad F = \frac{M_{e30}.V_c}{d_{30}} \quad F = \frac{V_c.(d_{30}.V_{30}^2 - G.Mv_{30})}{G_e.d_{30}}$$

$$V^2 = \frac{G.Mv + \frac{G_e.d.V_c.(d_{30}.V_{30}^2 - G.Mv_{30})}{G_e.d_{30}.V_c}}{d} \quad V^2 = \frac{G.Mv + \frac{d_{30}.V_{30}^2 - G.Mv_{30}}{d_{30}}}{d}$$

The Ge was cut, so that the speed does not depend on the gravitational constant of dark matter. Thus it is impossible that dark matter is radiated from the galaxy center just like the electromagnetic waves.

There were done 3 simulations by calculating indices from the speed in the outside orbit of 30Kpc, and the curve diverted from its path, as in the case of electromagnetic waves emitted by the stars, the curve has not diverted from its path.

DENSITY OF THE ELECTROMAGNETIC WAVES

Let's calculate the density of the electromagnetic waves from the sun that reaches Earth.

Sun power = 1367 w/m^2 in 1 m^2 of the area has 1367 w

Time for the light traverse 1 meter $t = \frac{1 \text{ metro}}{Vc}$ $t = \frac{1}{299792458}$

$t = 3.3356409 \times 10^{-9} \text{ s}$

$E = P.t$ $E = 1367 \times 3.3356409 \times 10^{-9}$ $E = 4.5598211 \times 10^{-6} \text{ J}$

$E = m.V^2c$ $m = \frac{E}{V^2c}$ $m = \frac{4.5598211 \times 10^{-6}}{299792458^2}$ $m = 5.0734852 \times 10^{-23} \text{ Kg}$

So in 1 m^3 has $5.0734852 \times 10^{-23} \text{ Kg}$ $\text{den} = 5.0734852 \times 10^{-23} \text{ Kg/m}^3$
or $\text{den} = 5.0734852 \times 10^{-26} \text{ g/cm}^3$

Then we will see the volume that is equal to the Earth, the mass of the electromagnetic waves from the sun that reaches the Earth:

Radius of the Earth = 6371 Km or 6371000 m $\text{Vol} = \frac{4\pi R^3}{3}$

$\text{Vol} = \frac{4}{3} \times \pi \times 6371000^3$ $\text{Vol} = 1.0832069 \times 10^{21} \text{ m}^3$

$m = \text{den} . \text{Vol}$ $m = 5.0734852 \times 10^{-23} \times 1.0832069 \times 10^{21}$ $m = 0.054956 \text{ Kg}$
 $m = 54.956 \text{ g}$

The electromagnetic waves are extremely tenuous, in a volume equal to the Planet Earth, the mass of the electromagnetic waves from the sun that reaches the Earth is only 55 grams.

Spreadsheet I (Nucleus)

R(m)	d(m)	S(w/m ³)	E(Joule)	E ^{total} (Joule)
9.2570400E+20	7.7142000E+18	1.0528015E-21	6.2508542E+48	3.5599477E+49
9.2570400E+20	1.5428400E+19	1.6843142E-22	7.9999611E+48	
9.2570400E+20	2.3142600E+19	5.3002566E-23	8.4958194E+48	
9.2570400E+20	3.0856800E+19	1.9655308E-23	7.4672759E+48	
9.2570400E+20	3.8571000E+19	1.0018535E-23	7.4329683E+48	
9.2570400E+20	4.6285200E+19	5.2438441E-24	6.7217932E+48	
9.2570400E+20	5.3999400E+19	4.4687661E-24	9.0946260E+48	
9.2570400E+20	6.1713600E+19	2.2244832E-24	6.7563407E+48	
$\Sigma +$				6.0219639E+49
9.2570400E+20	7.7142000E+18	1.6843142E-22	1.0000368E+48	
9.2570400E+20	1.5428400E+19	5.3002566E-23	2.5174547E+48	
9.2570400E+20	2.3142600E+19	1.9655308E-23	3.1505634E+48	
9.2570400E+20	3.0856800E+19	1.0018535E-23	3.8061558E+48	
9.2570400E+20	3.8571000E+19	5.2438441E-24	3.8905216E+48	
9.2570400E+20	4.6285200E+19	4.4687661E-24	5.7282637E+48	
9.2570400E+20	5.3999400E+19	2.2244832E-24	4.5271653E+48	
$\Sigma -$				2.4620161E+49
8.4856200E+20	7.7142000E+18	1.0528015E-21	5.7299345E+48	3.2630638E+49
8.4856200E+20	1.5428400E+19	1.6843142E-22	7.3332202E+48	
8.4856200E+20	2.3142600E+19	5.3002566E-23	7.7876493E+48	
8.4856200E+20	3.0856800E+19	1.9655308E-23	6.8447136E+48	
8.4856200E+20	3.8571000E+19	1.0018535E-23	6.8131041E+48	
8.4856200E+20	4.6285200E+19	5.2438441E-24	6.1610574E+48	
8.4856200E+20	5.3999400E+19	4.4687661E-24	8.3356601E+48	
8.4856200E+20	6.1713600E+19	2.2244832E-24	6.1922635E+48	
$\Sigma +$				5.5197603E+49
8.4856200E+20	7.7142000E+18	1.6843142E-22	9.1669798E+47	
8.4856200E+20	1.5428400E+19	5.3002566E-23	2.3076424E+48	
8.4856200E+20	2.3142600E+19	1.9655308E-23	2.8879478E+48	
8.4856200E+20	3.0856800E+19	1.0018535E-23	3.4888287E+48	
8.4856200E+20	3.8571000E+19	5.2438441E-24	3.5660759E+48	
8.4856200E+20	4.6285200E+19	4.4687661E-24	5.2504087E+48	
8.4856200E+20	5.3999400E+19	2.2244832E-24	4.1493637E+48	
$\Sigma -$				2.2566965E+49
7.7142000E+20	7.7142000E+18	1.0528015E-21	5.2090133E+48	2.9661566E+49
7.7142000E+20	1.5428400E+19	1.6843142E-22	6.6664712E+48	
7.7142000E+20	2.3142600E+19	5.3002566E-23	7.0794599E+48	
7.7142000E+20	3.0856800E+19	1.9655308E-23	6.2221211E+48	
7.7142000E+20	3.8571000E+19	1.0018535E-23	6.1931930E+48	
7.7142000E+20	4.6285200E+19	5.2438441E-24	5.6002603E+48	
7.7142000E+20	5.3999400E+19	4.4687661E-24	7.5765813E+48	
7.7142000E+20	6.1713600E+19	2.2244832E-24	5.6280764E+48	
$\Sigma +$				5.0175177E+49
7.7142000E+20	7.7142000E+18	1.6843142E-22	8.3335891E+47	

7.7142000E+20	1.5428400E+19	5.3002566E-23	2.0978276E+48
7.7142000E+20	2.3142600E+19	1.9655308E-23	2.6253251E+48
7.7142000E+20	3.0856800E+19	1.0018535E-23	3.1714862E+48
7.7142000E+20	3.8571000E+19	5.2438441E-24	3.2416055E+48
7.7142000E+20	4.6285200E+19	4.4687661E-24	4.7725014E+48
7.7142000E+20	5.3999400E+19	2.2244832E-24	3.7715059E+48

Σ^- 2.0513611E+49

6.9427800E+20	7.7142000E+18	1.0528015E-21	4.6880900E+48	2.6692185E+49
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6.9427800E+20	1.5428400E+19	1.6843142E-22	5.9997115E+48
6.9427800E+20	2.3142600E+19	5.3002566E-23	6.3712448E+48
6.9427800E+20	3.0856800E+19	1.9655308E-23	5.5994883E+48
6.9427800E+20	3.8571000E+19	1.0018535E-23	5.5732191E+48
6.9427800E+20	4.6285200E+19	5.2438441E-24	5.0393815E+48
6.9427800E+20	5.3999400E+19	4.4687661E-24	6.8173517E+48
6.9427800E+20	6.1713600E+19	2.2244832E-24	5.0637429E+48

Σ^+ 4.5152230E+49

6.9427800E+20	7.7142000E+18	1.6843142E-22	7.5001950E+47
6.9427800E+20	1.5428400E+19	5.3002566E-23	1.8880094E+48
6.9427800E+20	2.3142600E+19	1.9655308E-23	2.3626928E+48
6.9427800E+20	3.0856800E+19	1.0018535E-23	2.8541231E+48
6.9427800E+20	3.8571000E+19	5.2438441E-24	2.9171024E+48
6.9427800E+20	4.6285200E+19	4.4687661E-24	4.2945245E+48
6.9427800E+20	5.3999400E+19	2.2244832E-24	3.3935731E+48

Σ^- 1.8460045E+49

6.1713300E+20	7.7142000E+18	1.0528015E-21	4.1671435E+48	2.3722263E+49
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6.1713300E+20	1.5428400E+19	1.6843142E-22	5.3329110E+48
6.1713300E+20	2.3142600E+19	5.3002566E-23	5.6629667E+48
6.1713300E+20	3.0856800E+19	1.9655308E-23	4.9767759E+48
6.1713300E+20	3.8571000E+19	1.0018535E-23	4.9531348E+48
6.1713300E+20	4.6285200E+19	5.2438441E-24	4.4783683E+48
6.1713300E+20	5.3999400E+19	4.4687661E-24	6.0578849E+48
6.1713300E+20	6.1713600E+19	2.2244832E-24	4.4991856E+48

Σ^+ 4.0128371E+49

6.1713300E+20	7.7142000E+18	1.6843142E-22	6.6667639E+47
6.1713300E+20	1.5428400E+19	5.3002566E-23	1.6781784E+48
6.1713300E+20	2.3142600E+19	1.9655308E-23	2.1000371E+48
6.1713300E+20	3.0856800E+19	1.0018535E-23	2.5367195E+48
6.1713300E+20	3.8571000E+19	5.2438441E-24	2.5925414E+48
6.1713300E+20	4.6285200E+19	4.4687661E-24	3.8164331E+48
6.1713300E+20	5.3999400E+19	2.2244832E-24	3.0155222E+48

Σ^- 1.6406108E+49

5.3999400E+20	7.7142000E+18	1.0528015E-21	3.6462334E+48	2.0751962E+49
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5.3999400E+20	1.5428400E+19	1.6843142E-22	4.6661412E+48
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5.3999400E+20	2.3142600E+19	5.3002566E-23	4.9546930E+48
5.3999400E+20	3.0856800E+19	1.9655308E-23	4.3540326E+48
5.3999400E+20	3.8571000E+19	1.0018535E-23	4.3329754E+48
5.3999400E+20	4.6285200E+19	5.2438441E-24	3.9172378E+48
5.3999400E+20	5.3999400E+19	4.4687661E-24	5.2981803E+48
5.3999400E+20	6.1713600E+19	2.2244832E-24	3.9343835E+48

$\Sigma + 3.5103877E+49$

5.3999400E+20	7.7142000E+18	1.6843142E-22	5.8333909E+47
5.3999400E+20	1.5428400E+19	5.3002566E-23	1.4683570E+48
5.3999400E+20	2.3142600E+19	1.9655308E-23	1.8373831E+48
5.3999400E+20	3.0856800E+19	1.0018535E-23	2.2193002E+48
5.3999400E+20	3.8571000E+19	5.2438441E-24	2.2679411E+48
5.3999400E+20	4.6285200E+19	4.4687661E-24	3.3382418E+48
5.3999400E+20	5.3999400E+19	2.2244832E-24	2.6373529E+48

$\Sigma - 1.4351915E+49$

4.7303700E+20	7.7142000E+18	1.0528015E-21	3.1940762E+48	1.8173002E+49
4.7303700E+20	1.5428400E+19	1.6843142E-22	4.0873568E+48	
4.7303700E+20	2.3142600E+19	5.3002566E-23	4.3398480E+48	
4.7303700E+20	3.0856800E+19	1.9655308E-23	3.8133944E+48	
4.7303700E+20	3.8571000E+19	1.0018535E-23	3.7945273E+48	
4.7303700E+20	4.6285200E+19	5.2438441E-24	3.4299822E+48	
4.7303700E+20	5.3999400E+19	4.4687661E-24	4.6383993E+48	
4.7303700E+20	6.1713600E+19	2.2244832E-24	3.4437882E+48	

$\Sigma + 3.0741373E+49$

4.7303700E+20	7.7142000E+18	1.6843142E-22	5.1100116E+47
4.7303700E+20	1.5428400E+19	5.3002566E-23	1.2862232E+48
4.7303700E+20	2.3142600E+19	1.9655308E-23	1.6093758E+48
4.7303700E+20	3.0856800E+19	1.0018535E-23	1.9437307E+48
4.7303700E+20	3.8571000E+19	5.2438441E-24	1.9861097E+48
4.7303700E+20	4.6285200E+19	4.4687661E-24	2.9230061E+48
4.7303700E+20	5.3999400E+19	2.2244832E-24	2.3089240E+48

$\Sigma - 1.2568371E+49$

3.8571000E+20	7.7142000E+18	1.0528015E-21	2.6043504E+48	1.4807829E+49
3.8571000E+20	1.5428400E+19	1.6843142E-22	3.3324353E+48	
3.8571000E+20	2.3142600E+19	5.3002566E-23	3.5378169E+48	
3.8571000E+20	3.0856800E+19	1.9655308E-23	3.1080695E+48	
3.8571000E+20	3.8571000E+19	1.0018535E-23	3.0919409E+48	
3.8571000E+20	4.6285200E+19	5.2438441E-24	2.7940619E+48	
3.8571000E+20	5.3999400E+19	4.4687661E-24	3.7771028E+48	
3.8571000E+20	6.1713600E+19	2.2244832E-24	2.8031685E+48	

$\Sigma + 2.5048946E+49$

3.8571000E+20	7.7142000E+18	1.6843142E-22	4.1665445E+47
3.8571000E+20	1.5428400E+19	5.3002566E-23	1.0486620E+48
3.8571000E+20	2.3142600E+19	1.9655308E-23	1.3119531E+48

3.8571000E+20	3.0856800E+19	1.0018535E-23	1.5842186E+48
3.8571000E+20	3.8571000E+19	5.2438441E-24	1.6183660E+48
3.8571000E+20	4.6285200E+19	4.4687661E-24	2.3810794E+48
3.8571000E+20	5.3999400E+19	2.2244832E-24	1.8801838E+48

$\Sigma - 1.0241117E+49$

3.0856800E+20	7.7142000E+18	1.0528015E-21	2.0833865E+48	1.1832441E+49
3.0856800E+20	1.5428400E+19	1.6843142E-22	2.6654680E+48	
3.0856800E+20	2.3142600E+19	5.3002566E-23	2.8291050E+48	
3.0856800E+20	3.0856800E+19	1.9655308E-23	2.4846589E+48	
3.0856800E+20	3.8571000E+19	1.0018535E-23	2.4707541E+48	
3.0856800E+20	4.6285200E+19	5.2438441E-24	2.2315986E+48	
3.0856800E+20	5.3999400E+19	4.4687661E-24	3.0149446E+48	
3.0856800E+20	6.1713600E+19	2.2244832E-24	2.2359811E+48	

$\Sigma + 2.0015897E+49$

3.0856800E+20	7.7142000E+18	1.6843142E-22	3.3330856E+47
3.0856800E+20	1.5428400E+19	5.3002566E-23	8.3877843E+47
3.0856800E+20	2.3142600E+19	1.9655308E-23	1.0491366E+48
3.0856800E+20	3.0856800E+19	1.0018535E-23	1.2664590E+48
3.0856800E+20	3.8571000E+19	5.2438441E-24	1.2932280E+48
3.0856800E+20	4.6285200E+19	4.4687661E-24	1.9017522E+48
3.0856800E+20	5.3999400E+19	2.2244832E-24	1.5007931E+48

$\Sigma - 8.1834559E+48$

2.3142600E+20	7.7142000E+18	1.0528015E-21	1.5623879E+48	8.8518210E+48
2.3142600E+20	1.5428400E+19	1.6843142E-22	1.9983224E+48	
2.3142600E+20	2.3142600E+19	5.3002566E-23	2.1199659E+48	
2.3142600E+20	3.0856800E+19	1.9655308E-23	1.8605780E+48	
2.3142600E+20	3.8571000E+19	1.0018535E-23	1.8485192E+48	
2.3142600E+20	4.6285200E+19	5.2438441E-24	1.6677610E+48	
2.3142600E+20	5.3999400E+19	4.4687661E-24	2.2502349E+48	
2.3142600E+20	6.1713600E+19	2.2244832E-24	1.6662947E+48	

$\Sigma + 1.4974064E+49$

2.3142600E+20	7.7142000E+18	1.6843142E-22	2.4995711E+47
2.3142600E+20	1.5428400E+19	5.3002566E-23	6.2883881E+47
2.3142600E+20	2.3142600E+19	1.9655308E-23	7.8616160E+47
2.3142600E+20	3.0856800E+19	1.0018535E-23	9.4835784E+47
2.3142600E+20	3.8571000E+19	5.2438441E-24	9.6754129E+47
2.3142600E+20	4.6285200E+19	4.4687661E-24	1.4212539E+48
2.3142600E+20	5.3999400E+19	2.2244832E-24	1.1201324E+48

$\Sigma - 6.1222430E+48$

1.5428400E+20	7.7142000E+18	1.0528015E-21	1.0413024E+48	5.8577508E+48
1.5428400E+20	1.5428400E+19	1.6843142E-22	1.3307303E+48	
1.5428400E+20	2.3142600E+19	5.3002566E-23	1.4097536E+48	
1.5428400E+20	3.0856800E+19	1.9655308E-23	1.2348064E+48	

1.5428400E+20	3.8571000E+19	1.0018535E-23	1.2236264E+48
1.5428400E+20	4.6285200E+19	5.2438441E-24	1.1004165E+48
1.5428400E+20	5.3999400E+19	4.4687661E-24	1.4789636E+48
1.5428400E+20	6.1713600E+19	2.2244832E-24	1.0901215E+48

$\Sigma + 9.9097208E+48$

1.5428400E+20	7.7142000E+18	1.6843142E-22	1.6659175E+47
1.5428400E+20	1.5428400E+19	5.3002566E-23	4.1875869E+47
1.5428400E+20	2.3142600E+19	1.9655308E-23	5.2278868E+47
1.5428400E+20	3.0856800E+19	1.0018535E-23	6.2939492E+47
1.5428400E+20	3.8571000E+19	5.2438441E-24	6.4046353E+47
1.5428400E+20	4.6285200E+19	4.4687661E-24	9.3776701E+47
1.5428400E+20	5.3999400E+19	2.2244832E-24	7.3620539E+47

$\Sigma - 4.0519700E+48$

9.5656080E+19	7.7142000E+18	1.0528015E-21	6.4508961E+47	3.5506865E+48
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9.5656080E+19	1.5428400E+19	1.6843142E-22	8.2239115E+47
9.5656080E+19	2.3142600E+19	5.3002566E-23	8.6764494E+47
9.5656080E+19	3.0856800E+19	1.9655308E-23	7.5548035E+47
9.5656080E+19	3.8571000E+19	1.0018535E-23	7.4274085E+47
9.5656080E+19	4.6285200E+19	5.2438441E-24	6.6120808E+47
9.5656080E+19	5.3999400E+19	4.4687661E-24	8.7741061E+47
9.5656080E+19	6.1713600E+19	2.2244832E-24	6.3654892E+47

$\Sigma + 6.0085145E+48$

9.5656080E+19	7.7142000E+18	1.6843142E-22	1.0320403E+47
9.5656080E+19	1.5428400E+19	5.3002566E-23	2.5879282E+47
9.5656080E+19	2.3142600E+19	1.9655308E-23	3.2175477E+47
9.5656080E+19	3.0856800E+19	1.0018535E-23	3.8507696E+47
9.5656080E+19	3.8571000E+19	5.2438441E-24	3.8876115E+47
9.5656080E+19	4.6285200E+19	4.4687661E-24	5.6347676E+47
9.5656080E+19	5.3999400E+19	2.2244832E-24	4.3676154E+47

$\Sigma - 2.4578280E+48$

6.1713600E+19	7.7142000E+18	1.0528015E-21	4.1542420E+47	2.1510818E+48
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6.1713600E+19	1.5428400E+19	1.6843142E-22	5.2663256E+47
6.1713600E+19	2.3142600E+19	5.3002566E-23	5.5019102E+47
6.1713600E+19	3.0856800E+19	1.9655308E-23	4.7205960E+47
6.1713600E+19	3.8571000E+19	1.0018535E-23	4.5447027E+47
6.1713600E+19	4.6285200E+19	5.2438441E-24	3.9280708E+47
6.1713600E+19	5.3999400E+19	4.4687661E-24	4.9942956E+47
6.1713600E+19	6.1713600E+19	2.2244832E-24	3.3812853E+47

$\Sigma + 3.6491428E+48$

6.1713600E+19	7.7142000E+18	1.6843142E-22	6.6461235E+46
6.1713600E+19	1.5428400E+19	5.3002566E-23	1.6572251E+47
6.1713600E+19	2.3142600E+19	1.9655308E-23	2.0403114E+47
6.1713600E+19	3.0856800E+19	1.0018535E-23	2.4061417E+47
6.1713600E+19	3.8571000E+19	5.2438441E-24	2.3787622E+47

6.1713600E+19	4.6285200E+19	4.4687661E-24	3.3474736E+47
6.1713600E+19	5.3999400E+19	2.2244832E-24	2.4860837E+47
		$\Sigma -$	1.4980610E+48

1.5428400E+19	7.7142000E+18	1.0528015E-21	9.8769648E+46	2.2579504E+47
1.5428400E+19	1.5428400E+19	1.6843142E-22	1.0000825E+47	
1.5428400E+19	2.3142600E+19	5.3002566E-23	5.6923232E+46	
1.5428400E+19	3.0856800E+19	1.9655308E-23	2.9503725E+46	
1.5428400E+19	3.8571000E+19	1.0018535E-23	1.9178297E+46	
1.5428400E+19	4.6285200E+19	5.2438441E-24	1.2172675E+46	
1.5428400E+19	5.3999400E+19	4.4687661E-24	1.2177491E+46	
1.5428400E+19	6.1713600E+19	2.2244832E-24	6.9552657E+45	

$\Sigma +$ 3.3568858E+47

1.5428400E+19	7.7142000E+18	1.6843142E-22	1.5801566E+46
1.5428400E+19	1.5428400E+19	5.3002566E-23	3.1470933E+46
1.5428400E+19	2.3142600E+19	1.9655308E-23	2.1109236E+46
1.5428400E+19	3.0856800E+19	1.0018535E-23	1.5038385E+46
1.5428400E+19	3.8571000E+19	5.2438441E-24	1.0038194E+46
1.5428400E+19	4.6285200E+19	4.4687661E-24	1.0373465E+46
1.5428400E+19	5.3999400E+19	2.2244832E-24	6.0617683E+45

$\Sigma -$ 1.0989355E+47

Spreadsheet II (Disk)

R(m)	d(m)	s(W/m ²)	E(Joule)	E ^{total} (Joule)
9.2570400E+20	9.5656080E+19	1.787276788E-04	1.5835931E+49	8.0904303E+49
9.2570400E+20	1.2497004E+20	1.160140194E-04	1.7522626E+49	
9.2570400E+20	1.5428400E+20	9.733208930E-05	2.2370531E+49	
9.2570400E+20	1.9285500E+20	7.725858210E-05	2.7671854E+49	
9.2570400E+20	2.3142600E+20	6.578966760E-05	3.3821805E+49	
9.2570400E+20	2.6999700E+20	4.485061180E-05	3.1261736E+49	
9.2570400E+20	3.0856800E+20	3.355230870E-05	3.0407564E+49	
9.2570400E+20	3.4713900E+20	3.063461340E-05	3.4955512E+49	
9.2570400E+20	3.8571000E+20	2.440613370E-05	3.4178461E+49	
9.2570400E+20	4.2428100E+20	1.421318520E-05	2.3924753E+49	
9.2570400E+20	4.7303700E+20	9.793931200E-06	2.0298783E+49	
$\Sigma + 2.9224956E+50$				
9.2570400E+20	6.1713600E+19	1.787276788E-04	6.5983125E+48	
9.2570400E+20	9.5656080E+19	1.160140194E-04	1.0279270E+49	
9.2570400E+20	1.2497004E+20	9.733208930E-05	1.4700928E+49	
9.2570400E+20	1.5428400E+20	7.725858210E-05	1.7756893E+49	
9.2570400E+20	1.9285500E+20	6.578966760E-05	2.3564011E+49	
9.2570400E+20	2.3142600E+20	4.485061180E-05	2.3057248E+49	
9.2570400E+20	2.6999700E+20	3.355230870E-05	2.3386602E+49	
9.2570400E+20	3.0856800E+20	3.063461340E-05	2.7763334E+49	
9.2570400E+20	3.4713900E+20	2.440613370E-05	2.7848528E+49	
9.2570400E+20	3.8571000E+20	1.421318520E-05	1.9904209E+49	
9.2570400E+20	4.2428100E+20	9.793931200E-06	1.6485916E+49	
$\Sigma - 2.1134525E+50$				
8.4856200E+20	9.5656080E+19	1.787276788E-04	1.4511336E+49	7.3766201E+49
8.4856200E+20	1.2497004E+20	1.160140194E-04	1.6053055E+49	
8.4856200E+20	1.5428400E+20	9.733208930E-05	2.0488041E+49	
8.4856200E+20	1.9285500E+20	7.725858210E-05	2.5330276E+49	
8.4856200E+20	2.3142600E+20	6.578966760E-05	3.0940111E+49	
8.4856200E+20	2.6999700E+20	4.485061180E-05	2.8576198E+49	
8.4856200E+20	3.0856800E+20	3.355230870E-05	2.7770153E+49	
8.4856200E+20	3.4713900E+20	3.063461340E-05	3.1889807E+49	
8.4856200E+20	3.8571000E+20	2.440613370E-05	3.1142735E+49	
8.4856200E+20	4.2428100E+20	1.421318520E-05	2.1769121E+49	
8.4856200E+20	4.7303700E+20	9.793931200E-06	1.8431656E+49	
$\Sigma + 2.6690249E+50$				
8.4856200E+20	6.1713600E+19	1.787276788E-04	6.0475997E+48	
8.4856200E+20	9.5656080E+19	1.160140194E-04	9.4194608E+48	
8.4856200E+20	1.2497004E+20	9.733208930E-05	1.3468005E+49	
8.4856200E+20	1.5428400E+20	7.725858210E-05	1.6262643E+49	
8.4856200E+20	1.9285500E+20	6.578966760E-05	2.1570037E+49	
8.4856200E+20	2.3142600E+20	4.485061180E-05	2.1092718E+49	
8.4856200E+20	2.6999700E+20	3.355230870E-05	2.1377577E+49	
8.4856200E+20	3.0856800E+20	3.063461340E-05	2.5355272E+49	
8.4856200E+20	3.4713900E+20	2.440613370E-05	2.5406128E+49	

8.4856200E+20 3.8571000E+20 1.421318520E-05 1.8136320E+49
 8.4856200E+20 4.2428100E+20 9.793931200E-06 1.5000527E+49
 $\Sigma - 1.9313629E+50$

7.7142000E+20 9.5656080E+19 1.787276788E-04 1.3186221E+49 6.6579843E+49
 7.7142000E+20 1.2497004E+20 1.160140194E-04 1.4582496E+49
 7.7142000E+20 1.5428400E+20 9.733208930E-05 1.8603607E+49
 7.7142000E+20 1.9285500E+20 7.725858210E-05 2.2984872E+49
 7.7142000E+20 2.3142600E+20 6.578966760E-05 2.8051528E+49
 7.7142000E+20 2.6999700E+20 4.485061180E-05 2.5881752E+49
 7.7142000E+20 3.0856800E+20 3.355230870E-05 2.5121054E+49
 7.7142000E+20 3.4713900E+20 3.063461340E-05 2.8806444E+49
 7.7142000E+20 3.8571000E+20 2.440613370E-05 2.8084740E+49
 7.7142000E+20 4.2428100E+20 1.421318520E-05 1.9593657E+49
 7.7142000E+20 4.7303700E+20 9.793931200E-06 1.6542002E+49
 $\Sigma + 2.4143837E+50$

7.7142000E+20 6.1713600E+19 1.787276788E-04 5.4967977E+48
 7.7142000E+20 9.5656080E+19 1.160140194E-04 8.5593151E+48
 7.7142000E+20 1.2497004E+20 9.733208930E-05 1.2234252E+49
 7.7142000E+20 1.5428400E+20 7.725858210E-05 1.4766849E+49
 7.7142000E+20 1.9285500E+20 6.578966760E-05 1.9572804E+49
 7.7142000E+20 2.3142600E+20 4.485061180E-05 1.9123492E+49
 7.7142000E+20 2.6999700E+20 3.355230870E-05 1.9361888E+49
 7.7142000E+20 3.0856800E+20 3.063461340E-05 2.2936537E+49
 7.7142000E+20 3.4713900E+20 2.440613370E-05 2.2949658E+49
 7.7142000E+20 3.8571000E+20 1.421318520E-05 1.6355463E+49
 7.7142000E+20 4.2428100E+20 9.793931200E-06 1.3501473E+49
 $\Sigma - 1.7485853E+50$

6.9427800E+20 9.5656080E+19 1.787276788E-04 1.1860414E+49 5.9325406E+49
 6.9427800E+20 1.2497004E+20 1.160140194E-04 1.3110614E+49
 6.9427800E+20 1.5428400E+20 9.733208930E-05 1.6716565E+49
 6.9427800E+20 1.9285500E+20 7.725858210E-05 2.0634318E+49
 6.9427800E+20 2.3142600E+20 6.578966760E-05 2.5153632E+49
 6.9427800E+20 2.6999700E+20 4.485061180E-05 2.3175198E+49
 6.9427800E+20 3.0856800E+20 3.355230870E-05 2.2455959E+49
 6.9427800E+20 3.4713900E+20 3.063461340E-05 2.5698712E+49
 6.9427800E+20 3.8571000E+20 2.440613370E-05 2.4995709E+49
 6.9427800E+20 4.2428100E+20 1.421318520E-05 1.7390211E+49
 6.9427800E+20 4.7303700E+20 9.793931200E-06 1.4619931E+49
 $\Sigma + 2.1581126E+50$

6.9427800E+20 6.1713600E+19 1.787276788E-04 4.9458764E+48
 6.9427800E+20 9.5656080E+19 1.160140194E-04 7.6987196E+48
 6.9427800E+20 1.2497004E+20 9.733208930E-05 1.0999390E+49
 6.9427800E+20 1.5428400E+20 7.725858210E-05 1.3268986E+49
 6.9427800E+20 1.9285500E+20 6.578966760E-05 1.7571186E+49

6.9427800E+20	2.3142600E+20	4.485061180E-05	1.7147917E+49
6.9427800E+20	2.6999700E+20	3.355230870E-05	1.7337141E+49
6.9427800E+20	3.0856800E+20	3.063461340E-05	2.0503198E+49
6.9427800E+20	3.4713900E+20	2.440613370E-05	2.0473776E+49
6.9427800E+20	3.8571000E+20	1.421318520E-05	1.4556531E+49
6.9427800E+20	4.2428100E+20	9.793931200E-06	1.1983136E+49
		$\Sigma -$	1.5648586E+50

6.1713300E+20	9.5656080E+19	1.787276788E-04	1.0533599E+49	5.1968736E+49
6.1713300E+20	1.2497004E+20	1.160140194E-04	1.1636847E+49	
6.1713300E+20	1.5428400E+20	9.733208930E-05	1.4825837E+49	
6.1713300E+20	1.9285500E+20	7.725858210E-05	1.8276499E+49	
6.1713300E+20	2.3142600E+20	6.578966760E-05	2.2242570E+49	
6.1713300E+20	2.6999700E+20	4.485061180E-05	2.0451435E+49	
6.1713300E+20	3.0856800E+20	3.355230870E-05	1.9767943E+49	
6.1713300E+20	3.4713900E+20	3.063461340E-05	2.2555676E+49	
6.1713300E+20	3.8571000E+20	2.440613370E-05	2.1861059E+49	
6.1713300E+20	4.2428100E+20	1.421318520E-05	1.5144859E+49	
6.1713300E+20	4.7303700E+20	9.793931200E-06	1.2647716E+49	
		$\Sigma +$	1.8994404E+50	

6.1713300E+20	6.1713600E+19	1.787276788E-04	4.3947696E+48
6.1713300E+20	9.5656080E+19	1.160140194E-04	6.8374701E+48
6.1713300E+20	1.2497004E+20	9.733208930E-05	9.7629462E+48
6.1713300E+20	1.5428400E+20	7.725858210E-05	1.1768196E+49
6.1713300E+20	1.9285500E+20	6.578966760E-05	1.5563382E+49
6.1713300E+20	2.3142600E+20	4.485061180E-05	1.5163367E+49
6.1713300E+20	2.6999700E+20	3.355230870E-05	1.5299521E+49
6.1713300E+20	3.0856800E+20	3.063461340E-05	1.8048931E+49
6.1713300E+20	3.4713900E+20	2.440613370E-05	1.7969767E+49
6.1713300E+20	3.8571000E+20	1.421318520E-05	1.2731032E+49
6.1713300E+20	4.2428100E+20	9.793931200E-06	1.0435923E+49
		$\Sigma -$	1.3797530E+50

5.3999400E+20	9.5656080E+19	1.787276788E-04	9.2055160E+48	4.4444355E+49
5.3999400E+20	1.2497004E+20	1.160140194E-04	1.0160564E+49	
5.3999400E+20	1.5428400E+20	9.733208930E-05	1.2930032E+49	
5.3999400E+20	1.9285500E+20	7.725858210E-05	1.5908426E+49	
5.3999400E+20	2.3142600E+20	6.578966760E-05	1.9312566E+49	
5.3999400E+20	2.6999700E+20	4.485061180E-05	1.7702472E+49	
5.3999400E+20	3.0856800E+20	3.355230870E-05	1.7045723E+49	
5.3999400E+20	3.4713900E+20	3.063461340E-05	1.9358782E+49	
5.3999400E+20	3.8571000E+20	2.440613370E-05	1.8654887E+49	
5.3999400E+20	4.2428100E+20	1.421318520E-05	1.2831410E+49	
5.3999400E+20	4.7303700E+20	9.793931200E-06	1.0588362E+49	
		$\Sigma +$	1.6369874E+50	
5.3999400E+20	6.1713600E+19	1.787276788E-04	3.8434708E+48	

5.3999400E+20	9.5656080E+19	1.160140194E-04	5.9753974E+48
5.3999400E+20	1.2497004E+20	9.733208930E-05	8.5243916E+48
5.3999400E+20	1.5428400E+20	7.725858210E-05	1.0263378E+49
5.3999400E+20	1.9285500E+20	6.578966760E-05	1.3546845E+49
5.3999400E+20	2.3142600E+20	4.485061180E-05	1.3165903E+49
5.3999400E+20	2.6999700E+20	3.355230870E-05	1.3243048E+49
5.3999400E+20	3.0856800E+20	3.063461340E-05	1.5563434E+49
5.3999400E+20	3.4713900E+20	2.440613370E-05	1.5422849E+49
5.3999400E+20	3.8571000E+20	1.421318520E-05	1.0863882E+49
5.3999400E+20	4.2428100E+20	9.793931200E-06	8.8417866E+48

$\xi - 1.1925439E+50$

4.7303700E+20	9.5656080E+19	1.787276788E-04	8.0510925E+48	3.7641904E+49
4.7303700E+20	1.2497004E+20	1.160140194E-04	8.8759815E+48	
4.7303700E+20	1.5428400E+20	9.733208930E-05	1.1278152E+49	
4.7303700E+20	1.9285500E+20	7.725858210E-05	1.3840181E+49	
4.7303700E+20	2.3142600E+20	6.578966760E-05	1.6745590E+49	
4.7303700E+20	2.6999700E+20	4.485061180E-05	1.5284354E+49	
4.7303700E+20	3.0856800E+20	3.355230870E-05	1.4638451E+49	
4.7303700E+20	3.4713900E+20	3.063461340E-05	1.6511836E+49	
4.7303700E+20	3.8571000E+20	2.440613370E-05	1.5771912E+49	
4.7303700E+20	4.2428100E+20	1.421318520E-05	1.0721223E+49	
4.7303700E+20	4.7303700E+20	9.793931200E-06	8.6412051E+48	

$\xi + 1.4035998E+50$

4.7303700E+20	6.1713600E+19	1.787276788E-04	3.3646609E+48
4.7303700E+20	9.5656080E+19	1.160140194E-04	5.2260490E+48
4.7303700E+20	1.2497004E+20	9.733208930E-05	7.4466675E+48
4.7303700E+20	1.5428400E+20	7.725858210E-05	8.9521765E+48
4.7303700E+20	1.9285500E+20	6.578966760E-05	1.1785628E+49
4.7303700E+20	2.3142600E+20	4.485061180E-05	1.1415926E+49
4.7303700E+20	2.6999700E+20	3.355230870E-05	1.1434077E+49
4.7303700E+20	3.0856800E+20	3.063461340E-05	1.3365497E+49
4.7303700E+20	3.4713900E+20	2.440613370E-05	1.3154730E+49
4.7303700E+20	3.8571000E+20	1.421318520E-05	9.1849494E+48
4.7303700E+20	4.2428100E+20	9.793931200E-06	7.3877122E+48

$\xi - 1.0271807E+50$

3.8571000E+20	9.5656080E+19	1.787276788E-04	6.5417603E+48	2.8018880E+49
3.8571000E+20	1.2497004E+20	1.160140194E-04	7.1934063E+48	
3.8571000E+20	1.5428400E+20	9.733208930E-05	9.1092266E+48	
3.8571000E+20	1.9285500E+20	7.725858210E-05	1.1112920E+49	
3.8571000E+20	2.3142600E+20	6.578966760E-05	1.3340805E+49	
3.8571000E+20	2.6999700E+20	4.485061180E-05	1.2051098E+49	
3.8571000E+20	3.0856800E+20	3.355230870E-05	1.1382799E+49	
3.8571000E+20	3.4713900E+20	3.063461340E-05	1.2594227E+49	
3.8571000E+20	3.8571000E+20	2.440613370E-05	1.1673837E+49	

3.8571000E+20	4.2428100E+20	1.421318520E-05	7.5402234E+48
3.8571000E+20	4.7303700E+20	9.793931200E-06	5.7241339E+48
		$\Sigma +$	1.0826444E+50
3.8571000E+20	6.1713600E+19	1.787276788E-04	2.7395567E+48
3.8571000E+20	9.5656080E+19	1.160140194E-04	4.2463255E+48
3.8571000E+20	1.2497004E+20	9.733208930E-05	6.0350402E+48
3.8571000E+20	1.5428400E+20	7.725858210E-05	7.2305643E+48
3.8571000E+20	1.9285500E+20	6.578966760E-05	9.4632242E+48
3.8571000E+20	2.3142600E+20	4.485061180E-05	9.0947909E+48
3.8571000E+20	2.6999700E+20	3.355230870E-05	9.0153095E+48
3.8571000E+20	3.0856800E+20	3.063461340E-05	1.0392956E+49
3.8571000E+20	3.4713900E+20	2.440613370E-05	1.0033630E+49
3.8571000E+20	3.8571000E+20	1.421318520E-05	6.7983895E+48
3.8571000E+20	4.2428100E+20	9.793931200E-06	5.1957692E+48
		$\Sigma -$	8.0245556E+49

3.0856800E+20	9.5656080E+19	1.787276788E-04	5.2022505E+48	1.9276375E+49
3.0856800E+20	1.2497004E+20	1.160140194E-04	5.6948499E+48	
3.0856800E+20	1.5428400E+20	9.733208930E-05	7.1681560E+48	
3.0856800E+20	1.9285500E+20	7.725858210E-05	8.6503045E+48	
3.0856800E+20	2.3142600E+20	6.578966760E-05	1.0225604E+49	
3.0856800E+20	2.6999700E+20	4.485061180E-05	9.0309139E+48	
3.0856800E+20	3.0856800E+20	3.355230870E-05	8.2168810E+48	
3.0856800E+20	3.4713900E+20	3.063461340E-05	8.5011837E+48	
3.0856800E+20	3.8571000E+20	2.440613370E-05	7.4169953E+48	
3.0856800E+20	4.2428100E+20	1.421318520E-05	4.6423040E+48	
3.0856800E+20	4.7303700E+20	9.793931200E-06	3.4442587E+48	
		$\Sigma +$	7.8193701E+49	
3.0856800E+20	6.1713600E+19	1.787276788E-04	2.1863158E+48	
3.0856800E+20	9.5656080E+19	1.160140194E-04	3.3768356E+48	
3.0856800E+20	1.2497004E+20	9.733208930E-05	4.7777987E+48	
3.0856800E+20	1.5428400E+20	7.725858210E-05	5.6898148E+48	
3.0856800E+20	1.9285500E+20	6.578966760E-05	7.3661805E+48	
3.0856800E+20	2.3142600E+20	4.485061180E-05	6.9710730E+48	
3.0856800E+20	2.6999700E+20	3.355230870E-05	6.7559393E+48	
3.0856800E+20	3.0856800E+20	3.063461340E-05	7.5023443E+48	
3.0856800E+20	3.4713900E+20	2.440613370E-05	6.7727646E+48	
3.0856800E+20	3.8571000E+20	1.421318520E-05	4.3193703E+48	
3.0856800E+20	4.2428100E+20	9.793931200E-06	3.1988893E+48	
		$\Sigma -$	5.8917326E+49	

2.3142600E+20	9.5656080E+19	1.787276788E-04	3.8503958E+48	1.1231850E+49
2.3142600E+20	1.2497004E+20	1.160140194E-04	4.1712850E+48	
2.3142600E+20	1.5428400E+20	9.733208930E-05	5.1734311E+48	
2.3142600E+20	1.9285500E+20	7.725858210E-05	6.0622554E+48	
2.3142600E+20	2.3142600E+20	6.578966760E-05	6.7971377E+48	

2.3142600E+20 2.6999700E+20 4.485061180E-05 5.4268137E+48
 2.3142600E+20 3.0856800E+20 3.355230870E-05 4.5207357E+48
 2.3142600E+20 3.4713900E+20 3.063461340E-05 4.4809961E+48
 2.3142600E+20 3.8571000E+20 2.440613370E-05 3.8144689E+48
 2.3142600E+20 4.2428100E+20 1.421318520E-05 2.3477266E+48
 2.3142600E+20 4.7303700E+20 9.793931200E-06 1.7156113E+48

$\Sigma + 4.8360857E+49$

2.3142600E+20 6.1713600E+19 1.787276788E-04 1.6310467E+48
 2.3142600E+20 9.5656080E+19 1.160140194E-04 2.4993325E+48
 2.3142600E+20 1.2497004E+20 9.733208930E-05 3.4995761E+48
 2.3142600E+20 1.5428400E+20 7.725858210E-05 4.1064767E+48
 2.3142600E+20 1.9285500E+20 6.578966760E-05 5.1623232E+48
 2.3142600E+20 2.3142600E+20 4.485061180E-05 4.6337943E+48
 2.3142600E+20 2.6999700E+20 3.355230870E-05 4.0597468E+48
 2.3142600E+20 3.0856800E+20 3.063461340E-05 4.1276143E+48
 2.3142600E+20 3.4713900E+20 2.440613370E-05 3.5699419E+48
 2.3142600E+20 3.8571000E+20 1.421318520E-05 2.2213987E+48
 2.3142600E+20 4.2428100E+20 9.793931200E-06 1.6177565E+48

$\Sigma - 3.7129008E+49$

1.5428400E+20 9.5656080E+19 1.787276788E-04 2.4646528E+48 4.6559081E+48
 1.5428400E+20 1.2497004E+20 1.160140194E-04 2.5723617E+48
 1.5428400E+20 1.5428400E+20 9.733208930E-05 2.9795498E+48
 1.5428400E+20 1.9285500E+20 7.725858210E-05 2.9348490E+48
 1.5428400E+20 2.3142600E+20 6.578966760E-05 2.8513208E+48
 1.5428400E+20 2.6999700E+20 4.485061180E-05 2.1376685E+48
 1.5428400E+20 3.0856800E+20 3.355230870E-05 1.7216822E+48
 1.5428400E+20 3.4713900E+20 3.063461340E-05 1.6691181E+48
 1.5428400E+20 3.8571000E+20 2.440613370E-05 1.3982864E+48
 1.5428400E+20 4.2428100E+20 1.421318520E-05 8.5014556E+47
 1.5428400E+20 4.7303700E+20 9.793931200E-06 6.1383178E+47

$\Sigma + 2.2193467E+49$

1.5428400E+20 6.1713600E+19 1.787276788E-04 1.0705261E+48
 1.5428400E+20 9.5656080E+19 1.160140194E-04 1.5998321E+48
 1.5428400E+20 1.2497004E+20 9.733208930E-05 2.1581300E+48
 1.5428400E+20 1.5428400E+20 7.725858210E-05 2.3650555E+48
 1.5428400E+20 1.9285500E+20 6.578966760E-05 2.4991753E+48
 1.5428400E+20 2.3142600E+20 4.485061180E-05 1.9438232E+48
 1.5428400E+20 2.6999700E+20 3.355230870E-05 1.5991691E+48
 1.5428400E+20 3.0856800E+20 3.063461340E-05 1.5719654E+48
 1.5428400E+20 3.4713900E+20 2.440613370E-05 1.3297612E+48
 1.5428400E+20 3.8571000E+20 1.421318520E-05 8.1430771E+47
 1.5428400E+20 4.2428100E+20 9.793931200E-06 5.8581289E+47

$\Sigma - 1.7537558E+49$

9.5656080E+19 9.5656080E+19 1.787276788E-04 1.3039516E+48 1.3683228E+48

9.5656080E+19	1.2497004E+20	1.160140194E-04	1.0870936E+48
9.5656080E+19	1.5428400E+20	9.733208930E-05	1.0529558E+48
9.5656080E+19	1.9285500E+20	7.725858210E-05	9.4883318E+47
9.5656080E+19	2.3142600E+20	6.578966760E-05	8.8465637E+47
9.5656080E+19	2.6999700E+20	4.485061180E-05	6.4669921E+47
9.5656080E+19	3.0856800E+20	3.355230870E-05	5.1181919E+47
9.5656080E+19	3.4713900E+20	3.063461340E-05	4.8976736E+47
9.5656080E+19	3.8571000E+20	2.440613370E-05	4.0613577E+47
9.5656080E+19	4.2428100E+20	1.421318520E-05	2.4489615E+47
9.5656080E+19	4.7303700E+20	9.793931200E-06	1.7532656E+47

$\Sigma + 7.7521348E+48$

9.5656080E+19	6.1713600E+19	1.787276788E-04	6.3193386E+47
9.5656080E+19	9.5656080E+19	1.160140194E-04	8.4640875E+47
9.5656080E+19	1.2497004E+20	9.733208930E-05	9.1203712E+47
9.5656080E+19	1.5428400E+20	7.725858210E-05	8.3579701E+47
9.5656080E+19	1.9285500E+20	6.578966760E-05	8.0798040E+47
9.5656080E+19	2.3142600E+20	4.485061180E-05	6.0309439E+47
9.5656080E+19	2.6999700E+20	3.355230870E-05	4.8378942E+47
9.5656080E+19	3.0856800E+20	3.063461340E-05	4.6731160E+47
9.5656080E+19	3.4713900E+20	2.440613370E-05	3.9019026E+47
9.5656080E+19	3.8571000E+20	1.421318520E-05	2.3651771E+47
9.5656080E+19	4.2428100E+20	9.793931200E-06	1.6875148E+47

$\Sigma - 6.3838120E+48$

6.1713600E+19	9.5656080E+19	1.787276788E-04	5.0640168E+47	3.2843158E+47
6.1713600E+19	1.2497004E+20	1.160140194E-04	3.8349425E+47	
6.1713600E+19	1.5428400E+20	9.733208930E-05	3.5688899E+47	
6.1713600E+19	1.9285500E+20	7.725858210E-05	3.1236847E+47	
6.1713600E+19	2.3142600E+20	6.578966760E-05	2.8604501E+47	
6.1713600E+19	2.6999700E+20	4.485061180E-05	2.0650001E+47	
6.1713600E+19	3.0856800E+20	3.355230870E-05	1.6190625E+47	
6.1713600E+19	3.4713900E+20	3.063461340E-05	1.5379466E+47	
6.1713600E+19	3.8571000E+20	2.440613370E-05	1.2677263E+47	
6.1713600E+19	4.2428100E+20	1.421318520E-05	7.6062326E+46	
6.1713600E+19	4.7303700E+20	9.793931200E-06	5.4168603E+46	

$\Sigma + 2.6244029E+48$

6.1713600E+19	6.1713600E+19	1.787276788E-04	3.5015988E+47
6.1713600E+19	9.5656080E+19	1.160140194E-04	3.2871066E+47
6.1713600E+19	1.2497004E+20	9.733208930E-05	3.2173954E+47
6.1713600E+19	1.5428400E+20	7.725858210E-05	2.8328517E+47
6.1713600E+19	1.9285500E+20	6.578966760E-05	2.6599786E+47
6.1713600E+19	2.3142600E+20	4.485061180E-05	1.9500469E+47
6.1713600E+19	2.6999700E+20	3.355230870E-05	1.5448066E+47
6.1713600E+19	3.0856800E+20	3.063461340E-05	1.4782694E+47
6.1713600E+19	3.4713900E+20	2.440613370E-05	1.2252588E+47
6.1713600E+19	3.8571000E+20	1.421318520E-05	7.3827460E+46

6.1713600E+19 4.2428100E+20 9.793931200E-06 5.2412544E+46
 Σ^- 2.2959713E+48

1.5428400E+19 9.5656080E+19 1.787276788E-04 1.4468664E+46 4.6175078E+45

1.5428400E+19 1.2497004E+20 1.160140194E-04 1.0190670E+46

1.5428400E+19 1.5428400E+20 9.733208930E-05 9.0771819E+45

1.5428400E+19 1.9285500E+20 7.725858210E-05 7.6481595E+45

1.5428400E+19 2.3142600E+20 6.578966760E-05 6.8208819E+45

1.5428400E+19 2.6999700E+20 4.485061180E-05 4.8275012E+45

1.5428400E+19 3.0856800E+20 3.355230870E-05 3.7264222E+45

1.5428400E+19 3.4713900E+20 3.063461340E-05 3.4949908E+45

1.5428400E+19 3.8571000E+20 2.440613370E-05 2.8504046E+45

1.5428400E+19 4.2428100E+20 1.421318520E-05 1.6947313E+45

1.5428400E+19 4.7303700E+20 9.793931200E-06 1.1951348E+45

Σ^+ 6.5994742E+46

1.5428400E+19 6.1713600E+19 1.787276788E-04 1.2441964E+46

1.5428400E+19 9.5656080E+19 1.160140194E-04 9.3917626E+45

1.5428400E+19 1.2497004E+20 9.733208930E-05 8.5496493E+45

1.5428400E+19 1.5428400E+20 7.725858210E-05 7.2051284E+45

1.5428400E+19 1.9285500E+20 6.578966760E-05 6.5128023E+45

1.5428400E+19 2.3142600E+20 4.485061180E-05 4.6499813E+45

1.5428400E+19 2.6999700E+20 3.355230870E-05 3.6114070E+45

1.5428400E+19 3.0856800E+20 3.063461340E-05 3.4023741E+45

1.5428400E+19 3.4713900E+20 2.440613370E-05 2.7844063E+45

1.5428400E+19 3.8571000E+20 1.421318520E-05 1.6599650E+45

1.5428400E+19 4.2428100E+20 9.793931200E-06 1.1677946E+45

Σ^- 6.1377235E+46